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THE MATHEMATICS TEACHER

Volume XXXII Number 4

Edited by William David Reeve

The Mathematical Foundations of Architecture

By Mary E. Craver
The Rayen School, Youngstown, Ohio

The architecture of the ancient Greeks and the cathedrals of the mediaeval builders have always been looked upon with astonishment and wonder. How were such marvels of beauty designed? What knowledge, which we lack today, was able to produce such unequalled perfection? In architectural schools these ancient works serve as source material in studying the principles of design. The first exercises for a beginning student in design are studies of the ancient "orders," reproductions of parts of ancient buildings, and original problems adapting the Greek style. Of course students notice that certain proportions between parts produce pleasing results and they try by copying and reproducing the ancient examples to acquire a "feel" for such proportions. No teacher of design, however, ever suggests to a student the possibility that the Greeks had a formula for it. The student's method is a hit and miss, trial and error method—the method of experimentation. But the architecture of the ancients is too consistently beautiful to be the result of such haphazard method of arriving at a finished design. This is the thought of many serious students who are challenged by the consistent perfection in Greek architecture. Regardless of who the architect was, regardless of the number of buildings, we know today the same perfection exists. It almost seems as if this perfection is a divine revelation and the architect merely an agent for translating it into a physical existence. The same is true for the cathedrals in the Middle Ages. We have ample evidence from ancient writers and architects that design was the application of mathematical relationships developed into a science of building. Vitruvius, about 20 B.C., in his work, *De Architectura*, writes that the Greeks had fundamental measures which seem necessary in all buildings. These are taken from parts of the human body which was considered by them to be divinely proportioned. The height of the head is 1/8 of the height of the body. The foot is 1/6 of the same height, etc. The exact relation, however, between these measures and the modulus as used in their architecture is not known.

Viollet-le-Duc in his *Dictionnaire d'Architecture* writes that the building rose as the result of geometric construction; that the harmonious system used for the interior determines the visible proportions of the exterior; that the Gothic builders used by preference certain triangles, equilateral and isosceles.

The Italian architect Alberti (1404–1472) recognizes the use of the circle and of various polygons in the drawing of plans of buildings.

Leonardo da Vinci used the proportions

of the human body as related to the square and the circle.

Palladio, a little later, determined the height of a room by finding the mean proportion between its two sides. (Euclid's construction of the mean proportional.)

In 1509, Lucas Pacioli, in collaboration with Leonardo da Vinci, published his *De Devina Proportione.* This is the oldest known work which uses this term we know as the golden section or pentagon (Wolff, in his article in the Eleventh Year Book—see Bibliography—relates in detail of the relation of mathematics to architecture as evidenced in the writings of architects of different periods).

It is believed that the rules for building both in ancient and mediaeval times had been developed into a secret science. They were purposely kept secret in order to create respect for the temples or cathedrals which were considered the material images of the mystery of existence. Through this secrecy these rules became forgotten and there existed only in the centuries follow- the Middle Ages the use of the pentagram, the five-pointed star—it can be seen carved in some portion of the cathedrals as a protection and charm against evil powers.

Serious students, who have suspected the existence of some set of rules which governed the ancient and mediaeval builders, have been trying to rediscover the secret of such unmatched perfection.

Jay Hambidge of Yale offered his theory of Dynamic Symmetry. (An excellent description with drawings may be found in The Third Year Book—see Bibliography.) It was his belief that ancient Greek Design was based on dimensions which are incommensurable, that the ratios most commonly used were $1:\sqrt{2}$, $1:\sqrt{3}$, $1:\sqrt{5}$. The latter ratio, for example, when reduced to a rectangular area makes a rectangle whose sides are 1 and 5. He calls this a root-five rectangle. He analyzed 182 Greek vases in the Boston Museum. Eighteen fell in the root-two rectangle classification, six in the root-three, and the rest in the root-five class. His theory is not without foundation in Greek philosophy. To the Greeks, number was an expression of the spirit of things and in this thought the irrational number was more perfect than the rational. Geometrically speaking, the side of a square was considered as a rational number and the diagonal of a rectangle, formed by taking two squares, the irrational number which depended upon the square for its existence.

Griffith, in his *Natural System of Architecture,* published in London in 1845, maintains that the origin of pointed architecture lies in the adaptation of polygons. The triangle, for example, gives the architectural effect, the trefoil, which is used in the east end of Lincoln Cathedral. In the vegetable kingdom it is represented by Trifolium pratense. In each case of the use of a polygon he gives its use in architecture and its example in the vegetable kingdom. The exhibition of this close relationship between nature and geometry is very characteristic of mediaeval builders. The rose window of Saint Ouen de Rouen tells the story of the creation. And the pentagram or five pointed star is the center of the design of the window. This desire to tie nature and architecture together is also consistent with Pythagorean philosophy which sought to derive all nature from mathematical relations.

Another student, Frederick M. Lund, in 1921 published an intensive study far superior to any of the others of the relationship between geometry and architecture—see Bibliography. His theory, described in the following pages, is the result of long study undertaken in connection with a commission given him by his government (Norway) to rebuild the destroyed upper parts of the west elevation of the Cathedral of Nidaros in Norway. His theory was that the cathedral could be rebuilt as it was if he could only discover the geometric system upon which it was originally designed. Examining the proportions of the remaining parts of this elevation he discovered everywhere an angle of $63°26'$. This angle, as will be shown, is connected

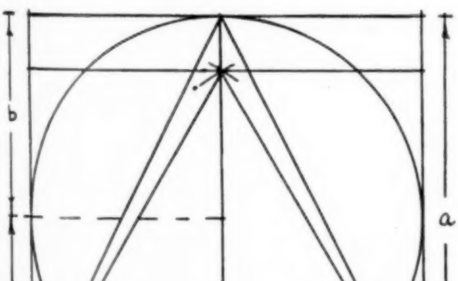
with the construction of the golden section or the pentagon.

Discovering this relationship in Nidaros he then set about to substantiate his theory. Did this or similar relationships exist in other cathedrals? If it did, could it be traced back to the ancient Greek Temples? And if found there, could it be traced still further to Egyptian architecture? He assembled all the working drawings, plans, elevations, sections, of innumerable examples of all these various periods. Where none existed he made drawings from his own measurements of the originals. Analyzing these working drawings, Lund found that the work of the ancient and mediaeval builders could be classified according to the principle of construction used.

One of the earliest systems in use he calls the *Construction ad quadratum*. See Plate I. This construction was the basis

PLATE I

The principles of construction
ad quadratum & ad triangulum.
ad quadratum 63°26′
ad triangulum 60°



∠63°26′ is connected with the construction of the sectio aurea and of the pentagon. This angle is the angle between the hypotenuse and the base of a rectangular triangle, when the length of the base is ½ the height, viz., when this same hypotenuse forms the diagonal in a rectangle composed of 2 squares and where the ratio between the base and height is as 1:2.

Lund—*Ad Quadratum*, p. 3.

for all religious architecture from the earliest times to the later Middle Ages. It is found in the Temple of Solomon, in ancient Greek architecture, in the three- and-five aisled churches of the thirteenth century. The application of this principle is shown on Plate II. The width of the temple is taken as the unit upon which the length is based. Vitruvius, in his rules for building a basilica, states that the ratio should be 1:2, 1:3.

In a three-aisled mediaeval church the nave gets two-fourths and each of the aisles one-fourth of the square. Each of the three aisles has its rectangle with the diagonal 63°26′, or occupying a space according to the ratio of 1:2 (Lund p. 25). In the five-aisled church, the nave often gets two sixths of the square. From this are produced five rectangles with a ratio between width and height of 1:3. The diagonal of the rectangle in this case forms an angle of 71°56′, approximately, with the base. This new diagnoal comes very near to the diagonal of the pentagon which forms with its side an angle of 72°. Therefore it might be said that the space in the five-aisled church is practically proportioned according to the *construction ad pentagonum*, another system explained later. But the construction ad quadratum, with the diagonal making an angle of 63°26′ with the base, is the leading principle for proportioning the entire building and architectural parts both for the three- and five-aisled church. See Plate III.

The Cathedral of Notre Dame, Paris, (1163–1235) was carried out ad quadratum in plan, transverse section, longitudinal section, and west elevation. The west elevation, the one usually shown in photographs, consists of a number of squares which combine in larger squares, and finally in one large square.

Cologne Cathedral, planned in 1248, is designed inside and out ad quadratum. It was designed by Albertus Magnus, a learned scholar of that city.

Other cathedrals using this construction are York, Durham (1093), Canterbury

(1096–1130), St. Mark's, Venice (1042–1071), Abbey Church of Cluny (1089–1131), Strassburg and Metz (both built in the thirteenth century).

Another principle of construction is that of *construction ad triangulum* as shown in Plate I, where the angle of 60° is obtained by constructing the equilateral triangle. The Cathedral of Notre Dame, Paris carried out in plan ad quadratum with

PLATE II

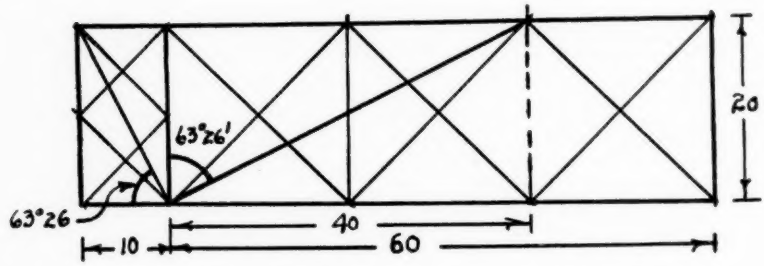


Fig. 1. Plan of Temple of Solomon. Measured in cubits. 1075 B.C.

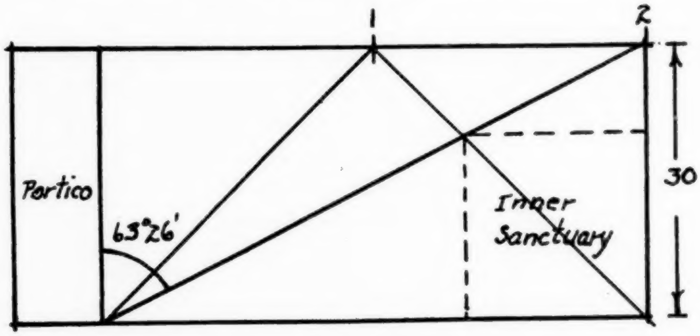


Fig. 2. Longitudinal Section of Temple of Solomon.

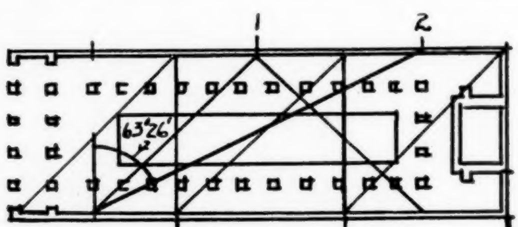


Fig. 3. Plan of Basilica of Pompeii—before 80 B.C.

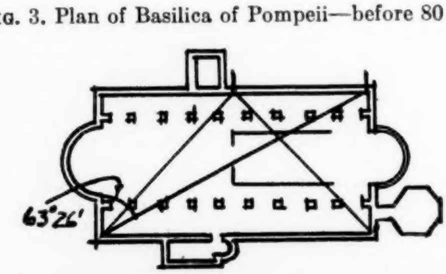
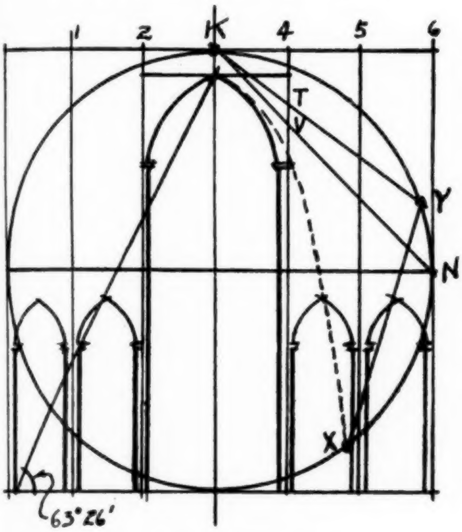


Fig. 4. Plan of the Anglo-Saxon Cathedral of Canterbury. Begun 1175.

Lund, pp. 5, 6

PLATE III

The five-aisled church:
The square-divided-by-six system



KN is the side of an inscribed square. KY, YX are sides of an inscribed polygon. T, V are points where sides of pentagon and square intersect outer wall of nave. Line TV represents that part of wall where thrust is greatest. The object of the flying buttress is to counteract this thrust. Whenever these arches have been correctly built they butt against the part of the wall between T and V. Thus the thrust of the vault is carried down in a line corresponding to the axis of the column. The line of thrust without abutment runs out from the column along the dotted line. This line passes through the point X which is the vertex of a base angle of the inscribed pentagon.

Lund, p. 26.

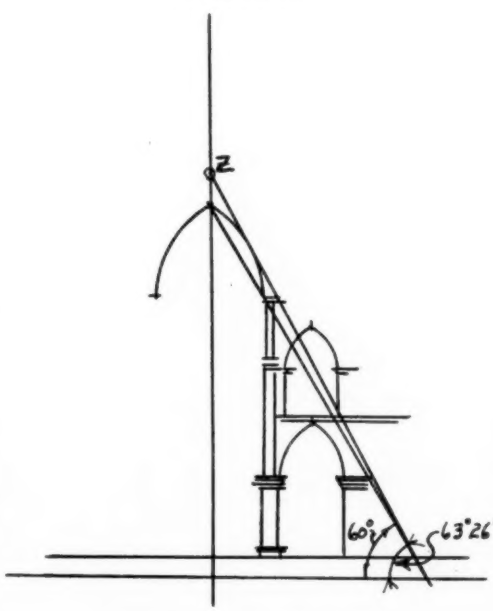
the ratio 1:3, is, however, built vertically ad triangulum. See Plate IV. If constructed ad quadratum the height of the vault would have been at Z. But it is much lower. At that time architects had not dared to rise too high in a church so broad.

Still another principle of construction was that of *construction ad pentagonum*, also known as sectio aurea. Plate V, Fig. 1 shows the German construction of the regular pentagon which was also used by Claudius Ptolemaeus in the first half of the second century A.D. Fig. 2 shows Euclid's Construction. In Book II, prop. 2, he states the problem: "To divide a given straight line in such a way that the rectangle enclosed by the line and one of the parts is equal to the square of the other part." Euclid's definition of the golden section: "A straight line is said to be divided in an extreme and a mean ratio when the whole length of the line is to the larger part as the larger is to the smaller." This geometric proportion is found seventeen times throughout Euclid's books II, IV, VI, VIII—the only proportion of all which enters constantly into a great number of proofs. See Plate VI. He thus calls attention to its importance. At the time of Pericles this proportion according to Lund is at its best in the Athenian buildings of the years 450–430. It occurs with such regularity that it cannot be a mere accident.

But the properties of the pentagon go back to the Pythagoreans themselves. From Pythagorus to Euclid, *i.e.* from about 569 through 300 B.C., the geometric and dynamic qualities of the pentagon or sectio aurea were looked upon with astonishment and admiration.

It was during this period, the time of

PLATE IV



Cathedral of Notre Dame (1160),
Transverse Section.

Lund, p. 36.

the Pythagoreans, that the Temple of Concordia (Doric) was erected. Jules Tannery, a French mathematician, says that in this Temple, "the length is equal to accurately four times the side of a decagon inscribed in a circle, the radius of which is equal to the width of the front." See Plate VII. Construction of the plan of the tem-

PLATE V

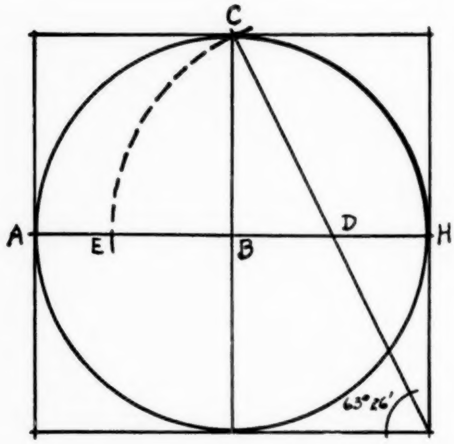


FIG. 1. German construction of the pentagon.

AB is the line to be divided. Given: radius *AB*, $BD=\frac{1}{2}BH$ or *AB*. With *D* as center, radius *CD*, cut *AB* at *E*. *EC* is the side of an equilateral and equiangular pentagon when it is used as a chord to divide the circle. *E* divides *AB* according to the proportion $AE/EB=EB/AB$. $\therefore EB$ is the mean between *AE* and *AB*. *EB* is also the side of a regular inscribed decagon.

Lund, p. 130.

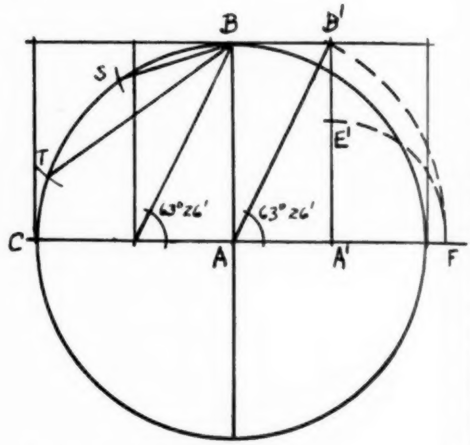


FIG. 2. Euclid's construction of the pentagon. Bk. II.

AB is the radius of a circle. *AA'* is ½ the radius. $AB \perp CA$. *AB'* is the 63°26' diagonal of a rectangle.

With *AB'* as radius, center *A*, draw arc *B'F*. With *A'F* as radius, center *A'*, draw arc *E'F*. *AB* is divided into *B'E'*, *E'A'*. In a circle with *AB* as radius, *A'E* will be the side of a decagon and *BT* (or *B'F*) the side of a pentagon.

PLATE VI

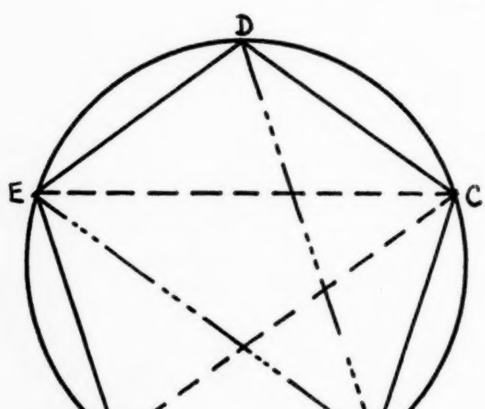


FIG. 1. Euclid's pentagon to illustrate Prop. 8. Bk. XIII.

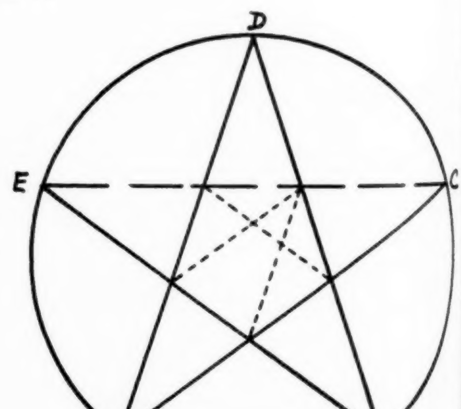


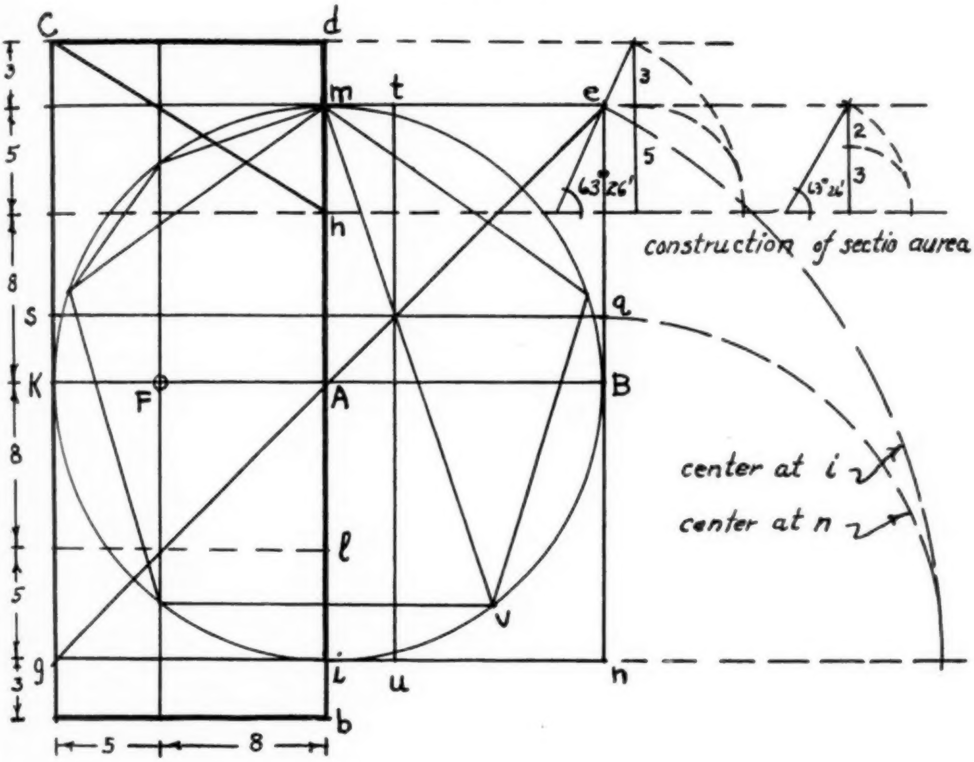
FIG. 2. Euclid's pentagon used in Bk. IV.

ABD is an isosceles triangle with $\angle A$, $\angle B$ twice $\angle D$. $\angle A$, $\angle B$ are bisected by *AC* and *BE*.

ple is as follows (see Lund, pp. 148–152):

Draw a circle having as radius AB the width of the front of the temple. Divide this radius, or AK, at F according to the sectio aurea. The segment AF, then, is the side of a decagon inscribed in the circle of the radius AB. This side of the decagon, which is the major of the radius, is carried

PLATE VII



Plan of Temple of Concordia showing system of construction ad pentagonum.
Lund, plate XII.

on the vertical diameter and is repeated four times on its extension ($Ah+hd$ and $Al+lb$). This gives the length of the temple bd. If the width of the front with its major is multiplied four times, the plan of the temple forms an irrational rectangle or heteromekeia. (A heteromekeia is a rectangle having one side the side of a square, and the other side a sectio aurea an irrational quantity from the square.)

On the sides of the center A of this heteromekeia, two squares or tetragons are produced, mK, Ag. The sides of the square, Am and Ai (=the radius of the circle) are divided according to the sectio aurea at the points h and l, and the major of the radius $Af=Ah=Al$, and the minor of the radius $FK=hm=li$.

The major ($dh=Ah$) of the radius is further divided according to the sectio aurea; the major of the radius' major is therefore hm (=the minor of the radius). The minor of the radius' major is dm.

If expressed in arithmetic values, as for instance, radius$=13$, its major ($Af=Ah=dh$) appears to be approximately 8 and its minor ($hm=FK$)$=h=5$, and the minor of the radius' major (dm)$=3$. The diameter is then 26. The length of the temple consists of the following arithmetic values in an harmonic, ascending and descending progression of the sectio aurea, 3:5:8 and

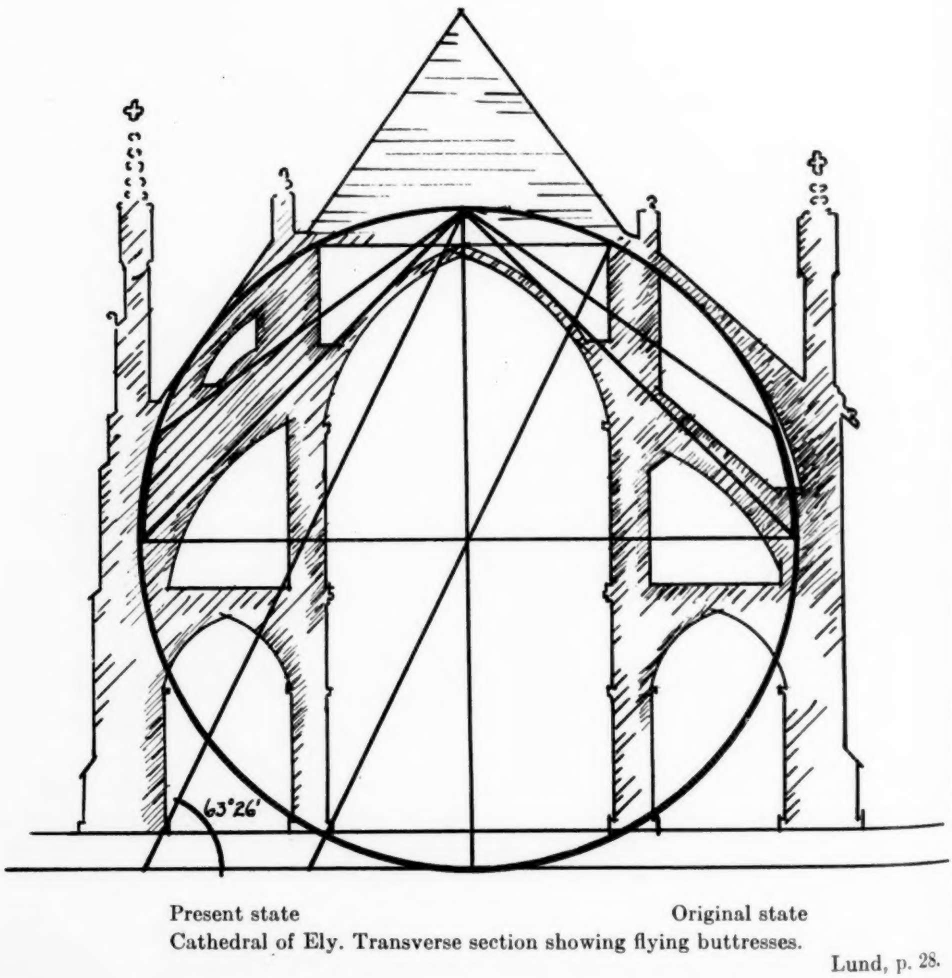
8:5:3. The plan or area consists of the heteromekeias of the sectio aurea: 3×13: 5×13:8×13: and 8×13:5×13:3×13. The temple plan consists of four equally large heteromekeias whose factors are equal to 8×13. The diagonals of these heteromekeias, such as *ch*, equal the side of the pentagon inscribed in the circle, having *AB* as radius.

In this plan we see the interplay between the large square, *ge*, and the temple heteromekeia where the side of the square is double the width (*KA*) of the temple heteromekeia.

Vitruvius, as stated before, said the Greeks had measures taken from the human body which were used in their buildings. A certain unit of measure or modulus was decided upon and then all the parts of a building from the diameter of the columns to the dimensions of the total width, length, and height of the building were harmoniously designed around this modulus or multiples of it. It has never been discovered just what the relation was between the measures taken from the human body and the modulus. In the Temple of Concordia a modulus was found equal to the radius of the circumference of the columns.

These principles of construction were used not only to produce designs of pleas-

PLATE VIII



Present state Original state

Cathedral of Ely. Transverse section showing flying buttresses.

Lund, p. 28.

ing proportions; they were used as part of the theory of construction itself since the science of mechanics was yet unknown. A system of abutment had to be developed to receive the thrust of the vault. See Plate VIII.

In the Cathedral of Ely, the Norman nave was build ad quadratum. The Gothic chancel (1235–1252) was built according to the same method. See Plate VIII. On the right side we see the abutment as originally built. The lower arch was built too low to receive the thrust. Consequently the thrust from the vault was in danger of pushing out the wall. In the fourteenth century this defect had to be repaired—as shown on the left of the drawing. It was necessary to lift the abutting point of the arch higher. So the flying buttress was lifted to that part of the wall which was included between these lines: the side of the pentagon and the side of the square which had been placed diagonally.

It is unfortunate that whatever science of building the ancient and mediaeval builders had has been lost. What we know now, or will know, will be acquired by such intensive research of which Lund offers an example. In the fourteenth century intellectualism declined and continued declining till by the eighteenth century all these fundamental constructions were lost. Goethe, writing in 1772 of Erwin von Steinbach, the architect of the Cathedral of Strassburg, was the first to arouse the feeling of appreciation of Gothic architecture. It has been only in comparatively recent times that interest has been aroused to the point of rediscovering these principles of construction.

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An Algebra Problem

THERE have been many arithmetic problems proposed similar to the one on p. 364 of the December (1938) MATHEMATICS TEACHER, but surprisingly no one suggests using these to demonstrate the usefulness and convenience of Algebra.

The problem: "Write down your age, multiply it by 2, add five and multiply the result by 50; add the amount of loose change from your pocket (less than \$1) and from the result subtract 365 and then add 115." The age will then appear in the hundreds column and the change in the units column.

This can be set down algebraically with no effort at all. Let a be the age and c the amount of change.

($2a$ plus 5)$\times 50$, plus c minus 365 plus 115; reducing, $100a$ plus 250, plus c minus 250.

Obviously the "250" cancels, leaving the age in the hundreds place ($100a$) and the change unaffected. Could these enticing problems be improved upon as material for elementary algebra?

E. E. ELY

Art in Geometry

By Lorella Ahern
Lincoln, Nebraska

Art appeals to the emotions. Its inception and final creation are both expressions of the emotions of the artist. Too, it is stimulating to the emotions of all those who enjoy the finished product.

The appeal of any art varies according to the medium. The particular medium employed may be words if the art is poetry, sound if the art is music, sound and a portrayal of emotions if the art is acting. It may be color and line in any of the many types of painting and design, marble if the art is sculpture, or coordination of line and time if the art is the dance. Though there are these mediums and many more, no true art can be produced by wild disordered work that ignores the underlying laws of order and control too subtle to be easily observed in the final expression. It is the recognition of these laws that distinguishes the work of the master from that of the artist controlled by whim only.

Mathematics is abstract. It is no more capable of being influenced by emotion than is an adding machine or a bridge. After all, these may be thought of as mathematics in action.

Great as the gap between mathematics and art may seem there is a striking relationship between the two. The importance of the role played by mathematics in art is not even suspected by many mathematicians, though some of the most able ones are aware of it and have studied and written about this relationship to some extent. Georg Wolff[1] of Düsseldorf, Germany, tells us,

> *Whether one has a narrow or broad conception of art, the mathematical method of working shows in its creative output, a close relationship to that of the artist. Whether the one works with brush, with chisel, or with pen, whether the other works with numbers, with relationships between quantities, with configurations in a plane, in space or in spaces, it is the creative imagination of each that is responsible for the finished product . . . Any one who has a sense of rhythm or an aesthetic sense must admit that mathematics and art are related to each other.*

This statement, or any statement similar to it, is apt to receive contempt, scorn, and denial from an artist. Though this is generally the case, artists must admit that the great artists of the Middle Ages were intensely interested in mathematics. This interest was not merely superficial. Mathematics to them was neither a pastime nor a dabbling in still another field while attempting to grasp all the knowledge of their time, as we are often led to believe. These artists studied mathematics because they realized that the advancement and refinement of their art was dependent, in various ways, upon mathematical knowledge.

Dr. Charles J. Martin,[2] Professor of Fine Arts, Columbia University, makes this point very clear in the following statement:

> *Are the aesthetic satisfactions in the good proportions of a work of art based upon geometric intervals? . . . It is a fact that the use of geometric measure and simple arithmetic progression in men's representation of beauty in architecture, sculpture, painting, and the minor arts, dates from the earliest known cultures. Clear evidence of it is to be found in a Greek sculpture as well as in Egyptian and Assyrian bas-reliefs in a Gothic cathedral, in a humble Peruvian tapestry weave, and in a Renaissance mural decoration. It is found even in pieces of jewelry carved by ancient craftsmen. An integral part of design, it is*

[1] Wolff, Georg. "Mathematics as Related to Other Great Fields of Knowledge," *National Council of Teachers of Mathematics*, Eleventh Year Book, pp. 226–27.

[2] Martin, Charles J., *Art Education Today*, Teachers College, Columbia University, p. 43.

marked outwardly by exquisite proportions, by precision on the whole effect, by unity, and by rhythmically recurring motifs.

Careful analysis reveals that it was consistently employed in painting by the immortal masters; and its use is also definitely observable in the compositions of many successful contemporary artists. How important an element it is in the construction of pictures may be appreciated by tracing it in the works of such men as Duccio, Giotto, Piero, Tintoretto, Titian, Giovanni Bellini, Fra Angelico, El Greco, Seurat, Cezanne, Leger, and Rivera—a list which though fragmentary seems imposing enough to stimulate an eager, but possibly skeptical student, to investigate reasons for an affirmative answer to the question at the beginning of this discussion. He will not find any written evidence, aside from some analysis of particular phases of the method by a few present day writers. The old masters, unfortunately, did not leave us treatises giving detailed procedures in their geometrical planning.

But written evidence is hardly necessary for reasonable conviction if one will merely study the works of art themselves, the new as well as the old.

However we must not leap to conclusions and assume that this geometric method will develop a formula for the production of a masterpiece. This is no more probable than that an ordinary writer will produce a literary masterpiece if we hand him a dictionary containing all the words necessary for the composition of any piece of literature.

Any one appreciative of mathematics cannot fail to notice that the most famous Florentine artists were serious students of mathematics. Only the comprehensive biographies of these artists show to what extent these men worked with mathematical problems. Less complete discussions of the lives and works of these masters usually pass over this point lightly, disposing of the entire matter in one or two terse sentences. This brevity may be due to fear that their readers will not be interested in a mathematical discussion, or, possibly to an erroneous idea that the artist's mathematical work concerned perspective only.

Perspective is of utmost importance to an artist, regardless of his period. In Italy great developments in perspective were due to Brunelleschi, the architect, and to Donatello, the sculptor.[3] Brunelleschi developed a correct technique of linear perspective, not accidentally, but by hardest study.[4] Over five hundred years ago, 1436, Leon Battisti Alberti[5] produced the first work upon this subject. He discusses the "Perspective image as the intersection of the pyramid of visual rays with the picture plane,"[6] and mentions an instrument by means of which he can construct it. Alberti's method was further developed by Piero della Francesca,[7] whom Mather tells us "Investigated with utmost zeal the mathematical basis of perspective."

The great German master Albrecht Dürer, celebrated for his painting, engravings, and wood cuts, too, studied mathematics in order to master his art. Cajori[8] tells us that Dürer discussed a new curve, the epicycloid, and gave the instrumental construction of it in his, "Underweysung der Messung mit dem Zyrkel und rychtscheyd," in 1525. His wood cuts show elaborate perspective drawings of spheres.[9] His well known painting "Melancholia" shows a magic square of 16 cells, and Cajori[10] states that Dürer is the earliest writer in the Occident to deal with magic squares. His scientific studies also included stereometric[11] investigations which he and Stifel had developed to some extent.

[3] Beman and Smith, *History of Mathematics*, Open Court Publishing Co., Chicago, p 226–27.

[4] Mather, Frank Jewett, Jr., *History of Italian Painting*, Henry Holt & Co., New York, p. 110.

[5] Beman and Smith, *op. cit.*, pp. 226–27.

[6] *Ibid.*

[7] *Ibid.*, p. 169.

[8] Cajori, F., *History of Mathematics*, Second Edition, Macmillan Co., p. 141.

[9] Albrecht Dürer, Dr. Franz von Jurafchef, Krystall Verlag Wien-Leipzig, p. 241.

[10] Cajori, *op. cit.*

[11] Beman and Smith, *op. cit.*, p. 224.

This famous artist is one of the founders of the modern theory of curves.[12]

Leonardo da Vinci is known to have had a deep interest in mathematics. This is shown both by his graphic works and by his writings. He said, "Let no man who is not mathematical read the elements of my work."[13] Leonardo's mechanical and scientific studies were so remarkable in compass and quality that they justified an investigation which resulted in such a work as "The Mechanical Investigations of Leonardo da Vinci," by Hart.[14] A characteristic of Leonardo is shown by a sheet of his drawings, now in the Uffizi, which consists of sketches of men's heads combined with studies of machinery.[15] One of his sketches for his most celebrated work *The Last Supper* shows geometric figures on the lower portion.[16]

When speaking of *The Adoration of the Kings*,[17] Mather states that, "In making such a masterpiece a clear and subtle geometry is involved." This art critic also states that Leonardo da Vinci's art can undergo mathematical demonstration[18] and that Leonardo "Brooded incessantly over mathematical and physical lore."[19] Leonardo's contemporaries admired and studied his composition and the geometric principles upon which it was based.[20]

We need not feel that an art which is subjected to a mathematical method of composition by the artist, or to a mathematical analysis for evaluation, will become formalized or mechanical. The early artists and architects realized that the beauty of the human figure was due to its proportion and symmetry. They studied the relation of parts of the body to the whole and to each other in order to construct buildings with the same degree of beauty. They knew that the buildings would possess this beauty if the symmetry of the building was the same as that of the human body.

The Roman architect, Vitruvius,[21] in his work *De Architectura*, says that "The planning of temples depends upon symmetry . . . for without symmetry and proportion no temple can have a regular plan; that is, it must have an exact proportion worked out after the fashion of the members of a finely-shaped human body." He then gives the proportions of the members of the human body and adds that, "By using these, ancient painters and famous sculptors have attained great and unbounded distinction."[22] This early ten volume work probably contains the first recorded canon. There followed many other canons including those that are in use today. Vitruvius also gives the proportions for the Corinthian and Doric columns. Here again is beauty based upon inflexible proportions, rather than upon something which the artist intuits. A rule for proportion does not lead to a stylized architecture, with each building a copy of the model. We see this when we contrast the Roman temples, of which Vitruvius wrote, with structures erected a thousand years later, on a different continent, by a primitive people under the guidance of a very few educated men who used Vitruvius' writings as their guide. These structures are the Southern California Missions, which bear little resemblance to the earlier ones based upon the same proportions, given by Vitruvius, for to quote Cheney,[23] "There is in the library of the Santa Barbara Mission on the Pacific Coast of America a copy of a Spanish edition of Vitruvius' *De Architectura* from which the

[12] *Ibid.*

[13] Hart, Ivor B., *The Mechanical Investigations of Leonardo da Vinci*, Open Court Publishing Co., Chicago, p. 8.

[14] *Ibid.*

[15] Mather, *op. cit.*, p. 231.

[16] Merycovski, Dmitri, *The Romance of Leonardo da Vinci.* Translated by Gurney, Hermitage Press, p. 68. Reproduction of this sketch.

[17] Mather, *op. cit.*, p. 236.

[18] *Ibid.*, p. 236.

[19] Mather, *op. cit.*, p. 240.

[20] *Ibid.*, p. 246.

[21] Granger, Frank, *Vitruvius, on Architecture*, G. P. Putnam's Sons, New York, Vol. I, Book II, p. 159.

[22] *Ibid.*, p. 161.

[23] Cheney, Sheldon, *A World History of Art*, The Viking Press, p. 484.

padres were able to transmit to their Amerindian workmen enough of the way of Roman architecture and column building to afford a touch of classic style to the mission church façade."

Michelangelo[24] is another master who recommended that architects study the proportions of the human body in order that they might produce buildings of beautiful proportions.

Architectural beauty was also dependent upon another phase of mathematics. The circle, which dominated Romanesque architecture; and the Gothic arch, which made possible the slender soaring beauty of Gothic structures, were both results of mathematical study. The circle was very confining when used as an arch or in cross-vaulting, for the width (a diameter) must always be twice the height (a radius). The Gothic arch gave a pliability which permitted greater height for the same width.

A notable achievement, made possible by the mathematical studies of the artist-architect Brunelleschi was the designing and the erection of the dome of the cathedral of Florence. In this structure the catenary was first used as the cross section of the dome.[25] Of this feat Gardner[26] says, "The daring ingenuity of Brunelleschi . . . made possible many beautiful later domes, of which the one at Florence was the prototype. Notably St. Peter's in Rome, St. Paul's in London, the Pantheon in Paris, and the U. S. Capital in Washington."

Not only the architects, artists and sculptors use mathematical laws to control their artistic output. During the fifth and sixth centuries B.C., the potters of Greece, a group of highly skilled technician artists labored on their exquisitely beautiful vases. These are things of enduring and superb beauty. How could they be otherwise when their creation was executed with such painstaking care and precision based on an elaborately organized knowledge of symmetry?

Only two peoples seem to have known and used this method of dynamic symmetry. At a very early date, possibly three or four thousand years B.C. the Egyptians developed a method of surveying. This led to the "cording of the temple." The method was then applied to the elevation plan and to the detail of ornament.[27] Some time during the sixth and seventh centuries B.C.[28] it fell into the hands of the Greeks. They refined it to an elaborately technical state. The purely mathematical concept has come down to us as geometry, but its application as a method of controlling area relationships in artistic fields was lost until Jay Hambidge restored it to us by his study about 1920.

Authentic fields for this research would be the bas-reliefs of Egypt, or Greek temples and theatres. However, few of these are left. The Greek vase, in all its aspects, is essentially the same as Greek architecture. Since there are hundreds of vases left, Hambidge used this material for his investigation. By this study he was able to return to the world the secret of dynamic symmetry.

Each of the vases analyzed by Hambidge was based on a unique pattern. Underlying the whole scheme of each of the vases, though they varied greatly, were the elements of dynamic symmetry, which are the logarithmic spiral, the root rectangle, reciprocal rectangles, poles, and whirling squares.

Though dynamic symmetry is now taught in many of the art schools in this country[29] some individuals feel that this dependence on rules, mathematical precision, and geometrically controlled lines and areas may hamper and stifle the artist. According to Miss Gisela A. M. Rich-

[24] Wolff, *op. cit.*, p. 239.

[25] Wolff, *op. cit.*, pp. 234–35.

[26] Gardner, Helen, *Art Through The Ages*, Harcourt Brace and Co., New York, p. 237.

[27] Hambidge, Jay, *Dynamic Symmetry, The Greek Vase*, Yale University Press, New Haven, Conn., p. 8.

[28] Beman and Smith, *op. cit.*, p. 16.

[29] Barnes, Albert C., *Art and Education*, Barnes Foundation Press, p. 293.

ter,[30] of the Metropolitan Museum, this is not the case. Concerning the Greek vase as analyzed by Hambidge and others she says:

"Each of these works of art forms a composition, in which the various details repeat the primary theme, thus producing a unified harmony. That is, the rectangles made by the lip, the neck, the body, the foot, and the handles, respectively bear a definite mathematical relation to the rectangle which contains the vase as a whole.

"We have an interplay of areas comparable to the sequence of phrases in a musical composition. It no more impedes the artist than harmony obstructs the composer or a metric system the poet. It merely supplies the law and order, which we know to be one of the dominant characteristics of Greek art."

Probably no one individual has done more to develop the present day standards of art than Louis Sullivan. Previous to a recent date the architecture in any American city had been a jumble of anachronisms. History gets rather mixed when modern railroad stations, apartment houses, and office buildings are housed in structures that closely resemble Greek temples, Venetian palaces, Gothic cathedrals, and feudal castles. Sullivan demanded an architecture based upon the building materials, the use to which the building was to be put, and upon the altered ways of human living evident in our machine age. His ideas were understood and adopted in Europe before they were in this country.

This change in architecture was the forerunner of a change in machine art. The taste and consequently the demands of the buying public have undergone a period of evolution which has resulted in our new and wholly satisfactory concept of machine art.

We need only to look about us to see clearly that we are living in the machine age of art. Cheney[31] tells us, "The American engineer was one of the two streams of influence that flowed together to produce industrial design, abstract art the other." The public is demanding this new commodity art, or industrial design. Mechanics and geometry are closely related, one deals with moving points, the other with static points.

Geometry of the high school level has been taught with questionable success from a functional viewpoint. It is usually presented in the traditional manner. This is splendid for those students who have a native ability to handle this work. However, they could do more than is covered in the usual class. The teacher would be happy to give them a richer course, but so much of her time and energy must be expended on the students lacking in mathematical ability, who consequently have no interest in geometry, that she is unable to do this. The more the teacher drives, though using every device of which a clever and well trained teacher is capable, the more the students dislike geometry. This is due, not to a lack of the teacher's skill, but to the fact that both teacher and pupil seldom have the emotional satisfaction of a personal achievement or contribution. Most geometry texts include all the traditional material of the past to satisfy the conservatives and most of the new material to attract the faddists. The book companies are compelled to do this in order to meet the demands of their customers. Any geometry course could be reduced to the barest essentials and be just as effective in the majority of cases as it is at present.

The English mathematicians, who keep their secondary level work on a plane far above ours, contend that the three congruency theorems may be postulated. This one item is a boon in time saving and in pupil interest conservation. So too, with many of our other formalized proofs. These proofs are subsequently used only

[30] Gugle, Marie, *Dynamic Symmetry*, National Council of Teachers of Mathematics, Third Year Book, p. 64.

[31] Cheney, Sheldon and Martha, *Art and the Machine*, Whittlesey House, p. 4.

from the standpoint of the information in the wording in the theorem itself, not from the stand point of information in their proof.

Such a procedure would leave ample time in the geometry course for an enrichment of geometry through its art applications. The student would not be deprived of anything he now gets under the traditional class set up, but he would have time to develop concepts and attitudes valuable to him. This enriched course would bring much joy to the teacher in its presentation. The type of student who at present accomplishes little, either through lack of interest or lack of ability, would in an enriched course accomplish much, both in factual material and in concepts, attitudes and appreciations. The work could be so full of satisfactions for both pupil and teacher that each could work on a higher level of enthusiasm and efficiency.

The girls in a geometry class may develop the ability to set a table, arrange furniture, select drapes for the home, detsign or choose clothes to a better advantage, all through concepts of harmony, line, symmetry, rhythm, and unity.

The boys may similarly develop a discrimination in the selection of window or show case displays. They study the designs of automobile bodies, and they study the arrangement of store and office fixtures. High school boys are capable of developing artistic concepts if they are given a little guidance. This they seldom get.

Such activities in the future lives of the pupils may be molded on something seen shallowly from the outside, or may give them a deeper insight, affording them a basic sense of a design for living, that will develop and extend from the creative center of a geometry class to an outward expression of personality.

In the actual procedure in my classes, the pupils are told that if they desire they may do some art work in their geometry class. They are told that it is a matter of individual work, probably unlike any art work they may have done previously. Each individual works on something he desires to do, only if he desires to do it. Emphasize over and over again that this work will constitute no part of their final mark. Explain that individuals differ in their graphic art ability just as they differ in musical ability or in mathematical ability. Least of all do we want to make artists of them, and perhaps their efforts will be feeble, but many of them take piano or vocal lessons with equally feeble efforts. They continue these lessons for personal enjoyment and for the development of appreciations, rather than for vocational plans.

After this talk is given, nothing more is done to introduce the art work until the students eventually request it. This, they have never failed to do.

When the students are asking for the art work the class room is turned into a laboratory for about three days. After this period the work is individual home work. Various art materials are placed in plain view and at the disposal of the students. When the work is more familiar to them the students bring materials especially adapted to their own piece of work.

The actual work may be introduced by designing borders. Overemphasize the simple ones to prevent any student from feeling that he may not attempt the work. Any of this work may be copied. The label should tell what it is copied from. This improves the ethics of the situation and puts a slight premium on original work.

Present very simple bridge designs for those individuals to whom borders may seem useless and deadly. Anyone not interested in borders or bridges may sketch his idea of a super modern airplane or a racing automobile. The instructor who knows her class will have no trouble in selecting items that will appeal to the enthusiasm and imagination of all students.

The work is highly individualized. It has no direct bearing on the day's formal geometry assignment, and is done when and if the student desires to do it. The tastes of each student shall dictate what

he shall work on. Some truly beautiful work may be done with designs for stained glass windows, styled in the Gothic tradition. Designs for leaded glass windows, and designs derived from magic square lines offer interesting work. Designs for furniture usually follow the modern trend for furniture design, which in its simplicity is good art and is distinctly geometrical in feeling. Other topics which are popular are floor plans for houses, and re-designing such everyday things as telephones, gas stoves, lighting fixtures, radios, linoleum, textiles, and wall paper.

We must have some knowledge of geometry in order to be aware of the geometry in nature. This geometry is apparent in crystals, in phyllotaxis, in flowers, in the cross sections of stems, and in the cross sections of seed pods. These geometric laws in nature are also seen in snow flakes, in the leaf distribution on plants which correspond to the Fibonacci series, and even to the angle and energy of a limb of a tree which branches from another limb.[32]

Color is one of the most important things in the world. All individuals are responsive to it in varying degrees. Color therapy is now used with success in the treatment of nervous disorders.[33] In this type of work it is only natural to let students use color. It is true that color is not geometry, but the field of art, exclusive of color, is so narrowed that we would be excluding much of the zest of the art which does apply to geometry. Neither is a pure geometry course concerned with stream-lined trains, with soaring skyscrapers, with flying buttresses, nor with the romance in the lives of forest rangers, or the navigators of the seven seas, but the basic functioning of all of these gives a richer appreciation of geometry.

The students will be interested in the lives and achievements of Louis Sullivan, Frank Lloyd Wright, Claude Bragdon, Howard Giles, and Bertram Goodhue. A little time may well be spent on analysis of some of the masterpieces of painting, from both the East and the West, for the Occidental prefers the equilateral triangle, while the Oriental definitely prefers the scalene. A few Oriental prints will show the beauty that is possible in the simple rhythm of curved lines.

The achievement of beauty and the achievement of the appreciation of beauty are two goals of life. These may well be reached through mathematics if we will but point the way.

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[32] Wolff, *op. cit.*, p. 221.

[33] Goldstein, Harriet & Vetta, *Art in Every Day Life*, Macmillan Co., New York, 1937, p. 207.

It Is Given Us To Dream

By Elmer Brill, *Chicago, Ill.*

The clod becomes alive
And dreams,
And then becomes a clod again.

But that dream
In the form of mathematics
Subdues nature.

Then dreams are effective—
Let us dream again
And conquer death!

The Educational Value of Logical Geometry

By J. H. Blackhurst
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The effort involved in attempting to substitute a living concept of education for the static preparation theory of Herbart has resulted in much progress in educational theory and some improvement in educational outcomes. Herbart's contribution to education left faculty discipline without followers except for those who did not know what it was all about. However, as might be expected, this made little or no difference at the time in either curriculum or method. Dewey moved beyond Herbart leaving faculty discipline still farther behind until at the present time faculty discipline is no longer scoffed in theory: it has been all but forgotten. Few educators could tell exactly what it is, or was thought to be. In spite of this fact we are still slaves to its clutches as far as the teaching of logical geometry is concerned.

Meanwhile educational theory has swung to the opposite extreme and is emphasizing education as a process of learning to live by living. Under this concept of education every effort is made to enrich the experience of the individual but little or nothing is said about improving the quality of this experience through some formal attention to the processes of reasoning designed to make the individual a critical thinker. In other words, while practice is still under the influence of faculty discipline, educational theory has forged ahead ignoring all mental discipline except that incidental to life-centered situations. Were such theory completely practiced the result would be a "sailor" mind in contrast to the mind of a disciplined thinker. There is need for a study of the scientific method of induction as such and also a need for a study of the processes of deduction. Such a discipline in reasoning might be called a discipline of processes in contrast to faculty discipline.

It is the writer's belief that the next step in education will emphasize discipline as a means of putting stiffening into "life-centered" education. If this happens a search will be made for a content which will lend itself to a study of the processes of reasoning. Such a content should be simple, non-contentious, and well-organized. This description of content points directly to logical geometry. No subject is more simple, none less contentious, and none better organized.

In other words, there is a compelling need in education which can adequately be met by no other subject than geometry. But to meet this need geometry must be freed from the fetters of faculty discipline and its content so organized and taught as to make it a source for exhibiting and studying the processes of rigorous reasoning.

To see clearly the value of demonstrative geometry it is necessary to accept the proposition that reasoning as a process may be improved. When we investigate the method by which improvement in reasoning as a process is to take place we find two possibilities. First, there is the possibility involved in the theory of faculty discipline. The mere exercise of rigorous reasoning has in the past been considered the method of improving the processes of reasoning. Second, there is what the writer chooses to call the "experience" theory of discipline or discipline of reasoning processes. We improve the processes of reasoning by attending to the processes of reasoning and thus generating meaning or experience with respect to the processes of reasoning.

Faculty Discipline Theory. The theory that mere exercise improves a process is scarcely tenable. Exercise as such may as readily cause a process to deteriorate as to improve. It is not the exercise of reasoning in playing chess which causes us to im-

prove in the game but rather the experience with the game of chess. Since attention is not on our reasoning process when playing chess but rather upon the meaning of the moves we may or may not make, we obtain no experience with reasoning when playing chess. Intelligence is engaged in what it is about, *i.e.*, the game, and experience is generated with respect to the game only. Our reasoning with respect to the game of chess is improved but our reasoning processes as such are not. Hence, the reasoning in chess, which does not result in experience with the process of reasoning, cannot transfer beyond chess.

Of course, if mind were some metaphysical thing existing antecedent to experience which might be improved as may a muscle by exercise the above conclusion would not be justified. But the nature of mind we assume is entirely expressed in the nature of meaning; it is the same thing. Mind follows the activity of intelligence rather than precedes it. Hence, to improve one's mind (experience) with respect to reasoning, attention must be focused upon the processes as such.

Strictly speaking reasoning, while it results in increased mind or experience, is not in itself mental. Reasoning itself manifests intelligence rather than mind. Reasoning as a process is as automatic and biological as walking. One may carry on reasoning all one's life without the slightest attention to it. When we reason, mind or meaning directs the reasoning act and the intelligence involved in it to the ends for which the reasoning is being done. Hence, unless time is set apart for a study of the reasoning processes no experience with reasoning can be generated.

Applied to geometry it is not only possible but highly probable that the pupil who has reasoned through all the material of a present day geometry has not in the least added to his experience with reasoning. In other words he has experienced geometry but not reasoning. We must then justify such a course of experience upon the grounds that geometric meaning alone justify the cost and effort involved. An attempt to do this would show the educational futility of geometric proof which at the same time gives no experience with the process of reasoning. The spatial facts of geometry in no way depend upon proof for their meaning. In fact the attempt to prove a proposition sometimes tends to detract from the meanings of the geometric facts themselves.

The value of demonstrative geometry lies in the fact that it may be utilized as object matter with which to stimulate and carry through a study of many of the reasoning processes. Unless such a study is made, experience will accumulate and will tend to improve future reasoning which operates upon past experience as content, but nothing will be coming from improved processes which will tend to guarantee an improved quality of future reasoning. A man may add to his experience, but unless he takes on the scientific method of inductive thinking and improved methods of deduction bulk alone is being added.

We have been all too slow to see the educational possibilities of geometry with the result that there is very real danger that geometry will pass out of education as did formal grammar. If it does, and some foremost educational thinkers predict that it will, only an educational revolution can bring it back in less than a century of time. The tragedy of such an event lies in the fact that geometry best meets the need for experience with thinking. While it is still in the schools it would be an easy matter to make the modifications in content and method which would save this subject for future generations and at the same time avoid the waste of teaching under a method which cannot, to say the least, yield a maximum of educational benefit.

The materials of geometry might be used to introduce the pupil to a formal study of the scientific method, the method of induction, as well as a formal study of the deductive method. There is no good reason why inductive proof should not be

used along with deductive proof. Tradition alone accounts for the exclusive use of deductive proof in geometry as it is taught today. The Greeks had not advanced to the point of the scientific method. The fact that inductive proof is necessary to allow the principle of contrast to operate in the study of the deductive method will make its use in connection with demonstrative geometry imperative if geometry is to realize its potentialities in the field of education.[1]

The assumptions underlying the value and methods of teaching demonstrative geometry should be the following:

1. *The present content and method of demonstrative geometry are based squarely upon the theory of faculty discipline.*
2. *Faculty discipline is wholly indefensible and absurd. (This is generally admitted by educational leaders.) The substance concept of mind, an immaterial thing continuous in time and space, whether coming from afar or generated by the body, will not much longer wear the face of probability.*
3. *No experience with reasoning is involved in proving a proposition unless the attention is turned upon the reasoning process as such.*
4. *A formal discipline of processes (not faculty discipline) is indispensable to the development of that aspect of reasoning which is not improved by the mere addition of experience with things other than the reasoning processes.*
5. *Demonstrative geometry is the best material available for exhibiting and studying (disciplining) the reasoning processes.*

There are equally definite assumptions underlying the content and method of present day demonstrative geometry. These assumptions have not been brought to light because we have never had any clear idea of an alternative. We have not realized that there has been anything to decide. Our method has rather unconsciously developed. Euclid was not preparing a high school textbook and, hence, it is absurd to think that the mere fact of making it easier without any change in principle in the light of educational psychology can possibly make it suited to the educational needs of pupils. To say the least, one building a castle and finding it when completed to be a perfect railway terminal, would have no more to explain than the one who should suspect that with out educational intent Euclid should have fashioned a perfect educational structure needing only a reduction in size.

The writer hopes that his suggestions for improving the teaching of geometry have at least the advantage of being clean-cut. He believes he has expressed a root principle which is either definitely right or definitely wrong and, hence, is charged with possibilities for the reconstruction of geometry. The conclusion can best be expressed in the words of Wolfgang Kohler in discussing gestalt psychology: "But I think somebody *should* have stated that radical principle, because it is of so much higher scientific value to make a clean, clear mistake, which is the best antecedent of progress, than to remain in that phase of vagueness where not even mistakes can be made—and afterwards be displaced by something better."

[1] By the principle of contrast is meant the part which the opposite plays in knowing anything. If we never experienced night we would not know day. If there was no illness in the world we would have health but we would not experience it. Similarly, if the child is to have experience with deduction he must also have experience with induction.

Vocabulary Instruction in Mathematics

By Richard M. Drake
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The attempt to provide more satisfactory study direction in secondary schools started with the administrative problem of securing the most appropriate time and place for study. The outcome was a widespread use of "supervised study" in the classroom procedure. This study activity in the presence of the teachers brought to their attention the need for analyzing those activities in which the pupils engage when they study and the need for the improvement of the pupil in the use of desirable study techniques.

Teachers have been slow in realizing the need for pupil diagnosis and remedial training in aspects of mathematics other than those which are strictly computational. Pupils have been required to memorize definitions with little thought being given by the teacher to the pupils' ability to understand words used in the definitions. Teachers in explaining mathematical problems often make use of many technical words, assuming that they are a definite part of the pupil's vocabulary.

One of the most difficult phases of any course in mathematics is the solution of "word" or "stated" problems. Apparently the importance of proper reading habits has been recognized, in part in connection with such problems. McCallister[1] says, "more attention has been given to interpreting problems than any other form of mathematical reading."

Modern textbooks in mathematics make much of the fact that they are "self-teaching." Obviously such books require detailed explanation of the procedures to be used. As a result the successful use of present day mathematics texts requires of the pupil an increased knowledge of the mathematical vocabulary. Humphrey[2] surveyed thirty-two text-books in junior high school mathematics and found that they contained 4,634 different words, 42% of which were mathematical terms. He concluded that "the large vocabulary used in junior high school texts must add greatly to the pupil's difficulty in solving problems."

Furthermore the recent textbooks have put added emphasis upon the "verbal" type of problem. Chateauneuf[3] surveyed 257 elementary algebra texts published since 1890. She found that one of the most significant changes since 1890 is the increase in both number and percentage of verbal exercises.

These two trends in textbook construction have placed an added burden upon the reading ability of the pupils. Frequently poor reading is due to a vocabulary deficiency. Brink[4] stressed this point by saying, "one of the basic reading needs of mathematics students is the acquisition of the vocabulary peculiar to the subjects which are being pursued. . . . Traditional practices in the teaching of arithmetic, algebra, and geometry stressed the learning of definitions which pupils were often expected to memorize verbatim long before they had any use for the terms. . . . Teachers often accept the glib recital of a formal definition as evidence that a pupil has acquired an understanding of a fundamental concept. Moreover, the

[1] J. W. McCallister, *Remedial and Corrective Instruction in Reading*, New York, D. Appleton-Century Co., 1936. Pp. 223–244.

[2] C. F. Humphrey, *The Vocabulary of Mathematical Textbooks in the Junior High School.* Unpublished Master's Thesis, University of Chicago, 1926.

[3] A. C. Chateauneuf, *Changes in the Content of Elementary Algebra Since the Beginning of the High-School Movement as Revealed by the Textbooks of the Period.* University of Pennsylvania, 1929.

[4] W. G. Brink, *Directing Study Activities*, New York, Doubleday, Doran and Co. Inc., 1937. P. 522.

statement of a formal definition is often unnecessary to a complete understanding of a process. . . . Since almost every page of a mathematics textbook contains several of these technical words or phrases, one of the first requisites in directing study is to make sure that every pupil understands the terminology used so that he may read the assignment intelligently."

In mathematics the vocabulary is of two types. The first is the technical vocabulary which is composed of words relating strictly to the subject. The second is a more generally functional type. It is comprised of words and expressions which are mathematical in nature but which function outside of the realm of mathematics. They are the words which are used to interpret mathematical concepts and appear most frequently in the verbal exercises as expressions of quantitative relationship.

In arithmetic the importance of both types of vocabulary has been recognized. Stephenson[5] asserted that, "very often a pupil fails to solve an arithmetic problem because one or more of the words are unfamiliar. Children do not understand as much about the meaning of words as their teachers give them credit for knowing. Not only are pupils deficient in general reading vocabulary but they are also unfamiliar with the many technical words used in arithmetics." Brueckner[6] in discussing the diagnosis of errors in arithmetic gave as one of his conclusions, "Vocabulary exercises on important arithmetical terms and number concepts are essential." Kinney[7] in summarizing the information that had been made available by studies in the field of problem solving stated that, "Considerable difficulty frequently results from the lack of a technical vocabulary."

The importance of a suitable vocabulary for the solution of verbal problems in algebra has also been recognized by several authorities in the field. Breslich[8] asserts that, "the vocabulary involved in verbal problems is a cause of serious difficulties. . . .Writers of textbooks on mathematics have not given the vocabulary the attention which it deserves . . . it is evident that the teacher must take the responsibility for teaching the meanings of the new words that occur in the problems. . . . Proper experiential background preceding the introduction of new words and technical terms is therefore necessary to eliminate the difficulty which arises from too limited a vocabulary." Schorling[9] maintains that special attention should be given to the teaching of reading in the solution of verbal problems. "The chances are high that most of the difficulties of pupils are due to low reading ability or low intelligence or to a combination of these two." Wren[10] surveyed the research in the teaching of secondary algebra. One of his conclusions was that the pupils' greatest difficulty with comprehension of problems is determining what is given and translating into algebraic symbolism."

The above opinions leave little doubt as to the importance of vocabulary in connection with problem solving in algebra. However, not as much emphasis has been placed upon the need for teaching a technical vocabulary in this field as in arithmetic. Perhaps it is because this type of vocabulary has little usage beyond the bounds of algebra. To the pupil studying algebra, however, this vocabulary is a very essential part of his experience. Over-

[5] P. R. Stephenson, "Difficulties in Problem Solving," *Journal of Educational Research*, 11: 95–103. Feb., 1925. P. 98.

[6] L. J. Brueckner, "Diagnosis in Arithmetic," *Thirty-fourth Yearbook of the National Society for the Study of Education*, 1935. Pp. 269–302.

[7] L. B. Kinney, "Problem Solving and the Language of Percentage," *The Journal of Business Education*, Jan., 1935. P. 24.

[8] E. R. Breslich, *Problems in Teaching Secondary School Mathematics*. Chicago, University of Chicago Press, 1931. P. 21.

[9] R. Schorling, *The Teaching of Mathematics*. Ann Arbor, Ann Arbor Press, 1936. P. 113.

[10] F. L. Wren, "Survey of Research in Teaching of Secondary Algebra," *Journal of Educational Research*, 38: 597–610, April, 1935.

man[11] in a talk before the National Council of Teachers of Mathematics at Cleveland, Ohio, said, "The learning of the algebraic language is, without doubt, the greatest difficulty presented to the beginner of the subject. . . . The pupils speak very glibly of 'canceling,' transposing, clearing of fractions, cross multiplying, etc., but do not understand what any of these terms really mean. Our lack of success in initiating pupils into the meaning of algebra has been partly, if not largely due to a lack of appreciation of the difficulties presented to the beginner by the notation or language of the subject." Lindquist[12] in summarizing the facts disclosed by the Iowa Every-Pupil Achievement Testing Program gave as one of his conclusions, "the language of algebra is almost totally incomprehensible to many algebra pupils." Authors of a few of the more recent texts in ninth grade algebra have recognized the need for more vocabulary teaching and have made a half way attempt to provide this instruction by inserting word lists at the beginning of each chapter. The use made of these lists, however, is left to the discretion of the teacher.

In conclusion, then, it is apparent that there is a decided need for diagnosis and remedial instruction in both the technical and the problem solving vocabulary of mathematics. The introduction of supervised study has brought the pupil and teacher in closer contact, with the resulting realization by the teacher of vocabulary deficiencies. The successful use of present day textbooks necessitates the understanding of words and expressions which appear in the "self-explanations" and the verbal problems. And, finally, while authorities within and outside of the field of mathematics have stated in emphatic language the importance of recognizing and meeting the situation, we must remember that these statements are opinions and are not based upon objective evidence. It still remains to be shown experimentally that instruction in the vocabulary of mathematics will improve the pupils' achievement in mathematics.

[11] J. R. Overman, "Teaching the Algebraic Language to Junior High School Pupils," THE MATHEMATICS TEACHER, 16: 216, April, 1923.

[12] E. F. Lindquist, "The Gap Between Promise and Fulfilment in Ninth Grade Algebra," *The School Review*, Dec., 1934. P. 768.

To the Entity of Mathematical Laws

By DAVID DE VRIES
New Brunswick High School Student

Eternal guide of all the Universe,
Thy power great I contemplate with awe
As thou thy wondrous acts doth e'er rehearse
And do thy endless work with ne'er a flaw.

The potent, perfect hand of God art thou,
And verily doth represent His mind;
If I to worship thee myself allow,
I shall a faultless deity thee find.

With consciousness of thee mere man must mate,
For how, if blind to thee, can he do aught?
If thou dost fail him, chaos is his fate,
And he and all his works must turn to naught.

In thee I see a deep intelligence
Which tells of strange things, mystic—weird—intense.

Meaningful Symbols as a Better Preparation for Junior and Senior High School Mathematics

By MYRTLE DUNCAN, *Training Teacher*
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IN THE tentative report entitled "Mathematics in General Education," The Commission on the Secondary School Curriculum of the Progressive Education Association emphasizes the importance of teaching symbolism in secondary mathematics and states that it seldom receives enough attention before the student reaches the high school level.

During the first six years of the child's life haven't we put too much emphasis on symbolism in mathematics? But isn't it symbolism without meaning? We need to emphasize the word, *meaningful* in connection with the symbolism that is taught before the child reaches the high school level. Many children who reach the seventh grade know the words—add, subtract, multiply, and divide, and the symbols for these words, but have no idea what process to use in solving a problem. They know the words and symbols for different units of measure but have little or no idea of their size. For example, they cannot estimate with any degree of accuracy distances of sixty feet, four yards, three-fourths of a mile. Then too most pupils do not understand common fractions or our number system well enough for the teacher to make a meaningful approach to decimals through either of these topics. Decimal fractions should be introduced either as a different and many times more convenient way of expressing a common fraction; for example, $\frac{3}{5}$ is .6, or as an extension of our number system. The child should get *both* of these concepts in connection with his study of decimals. Due to the teacher's lack of understanding of the relationship between common and decimal fractions and the fundamental structure of the number system, many children get neither of these fundamental ideas. Many children can use words or figures in writing fractions and decimals but do not see any meaning back of them; they know rules which they cannot apply. At the end of the sixth school year children ask the teacher how to express the remainder to every verbal problem in division which they encounter. The answer is not meaningful enough for them to know whether the remainder should be expressed as a whole number, common fraction, or decimal fraction, as in this problem: "Alice gathered 556 eggs and packed them in boxes of 24 eggs each. How many boxes did she need?" The remainder, four, in this problem means that Alice had four eggs left over and it should be expressed as a whole number.

Many children talk about percentage as if it were something different and isolated from anything that they have ever studied instead of knowing that it is a different way of writing common and decimal fractions. For example, 25% is .25 or $\frac{1}{4}$. Often pupils cannot solve problems in percentage which use such words as rate of discount, discount, and net proceeds, because they do not understand the situation with which the problem deals or know the meaning of the words which it uses. Mathematics for many children in the first six grades is one mass of abstractions or symbols without meaning.

Symbols, yes, but *meaningful* symbols! Such symbols as the digits, signs for processes, names of units of measure, rules, or any other symbols in mathematics should be preceded by meaning. Meaning for symbols can best be taught in connection with a felt need on the part of the child and in connection with a variety of situations with which he is familiar. William A. Brownell in his article entitled "Psychological Considerations in the Learning and Teaching of Arithmetic" in the *Tenth*

Yearbook of the National Council of Teachers of Mathematics tells us something of a meaningful approach to number concepts, number combinations, and fundamental processes in the primary grades. Some of the texts in arithmetic now help to emphasize the meaning of the fundamental processes by suggesting what question or questions each answers and by giving problems without numbers in which the child's attention is directed to the process rather than to the numbers, as, if you are given the cost of one article how will you get the cost of a certain number of such articles. Children are also encouraged to use a variety of verbal expressions in connection with the symbol for each process and to consider the reasonableness of their answers.

Primary teachers who take the attitude that deferring some topics of mathematics makes the work easier for them and solves all of the mathematical difficulties for the child are not helping the situation. Guy T. Buswell in his article in THE MATHEMATICS TEACHER for May, 1938, states: "Arithmetic should not be deferred beyond the primary grades; it should be properly selected and organized to build on the interests and needs which research has shown children possess at the age of entering school. This type of arithmetic should give meaning to normal social experiences. It is the school's best defense against abstract verbalism and meaningless formality in the middle and upper grades." If we agree with this statement, the responsibility of the primary teacher today for mathematics is greater than it has ever been before. To accomplish the ends suggested by Guy T. Buswell requires understanding of mathematics on the part of the teacher, good teaching, and systematic, not accidental, teaching of certain topics. The primary teacher should utilize the child's experience in teaching fractions in the primary grades and not assume that a child never uses fractions until he reaches the intermediate grades. The teacher needs to know the different meanings of a fraction and be sure in her teaching of them that the meaning is brought out to the child without his being required to make any formal statement regarding it.

Will the primary teacher fail to show the meaning of a fraction as an expression of ratio? Does she require the addition of fractions which no life situation would demand? Does the teacher force the child to use rules in computation with fractions without any understanding, meaning, or insight, instead of having him develop his rules? If she does, then there is no wonder that the child adds the denominators as well as the numerators in the addition of fractions. If the child is led through examination of specific cases to discover, and to make his own rules, they will have more meaning to him, and he will be better able to apply them.

William A. Brownell in his article previously mentioned suggests that in teaching meaning we show relationship between the different topics of the subject. He gives as an illustration the showing of relation between common fractions, decimal fractions, and per cent. Some of our new books in introducing percentage make use of this idea by having children compare their records in different situations with use of common fractions, decimal fractions, and per cents. An illustration from one text is as follows:

At target practice John hit the bull's-eye 21 times out of 25 trials; so he hit it 21/25 of the times. Tom hit it 17 out of 20 times; so he hit it 17/20 of the times. To decide which boy had the better score, they had to find out which fraction was larger, 21/25 or 17/20. Can you find out? A good way to tell which of the two fractions is larger is to express them as hundredths and then compare them.

In teaching certain applications of percentage, for instance discount, the child needs to be aided in his appreciation of the situation and taught the meaning of such expressions as discount, rate of discount, and net price. In my arithmetic

class last summer, the children through discussion took an imaginary visit around the square of the town in which they lived. In this discussion they got the meaning of such words as discount, rate of discount, and net price, saw why discounts were given, and solved problems containing data given to us by local merchants in different lines of business. Appreciation of situations and meanings of words are essential to a meaningful solution of problems dealing with any application of percentage.

In connection with meaningful symbols and the need for more comprehensive problems, it is well to review the following words so aptly stated by the commission previously mentioned in this discussion. "Our present courses probably do not provide adequate opportunities for the students to practice analyzing problem situations. Most of the problems which have been presented to students have been simplified and idealized to the point where all that remains to be done is to recognize and perform operations leading to numerical answers. The student is rarely given the opportunity to begin with a more comprehensive situation and go through the experience of simplifying and idealizing the problem for himself. Consequently he frequently fails to realize the many assumptions and restrictions which need to be made in order to treat quantitatively even the simplest situations." For instance, we give children problems of this type: If Mr. Horton's automobile averages 20 miles to the gallon of gasoline, and if gasoline costs 17 cents per gallon, how much will it cost him to drive 100 miles? The problems should be of this type: Mr. Horton is planning an automobile trip. How much will it cost him? The problem might be: we are planning a trip to ——— (in connection with English, history, or geography). What will it cost us?

As the last suggestion in my plea for teaching meaningful symbols in the first six grades, I recommend that every teacher of arithmetic of these grades study Frank McMurry's article in *Education* for April, 1934, entitled, "What Is the Matter with Arithmetic?" When the mathematics of the first six grades has meaning to the child, surely he will enter high school with a better foundation for continued work in this subject.

Please Renew May Expirations Now!

MEMBERS whose subscriptions will expire with the May number have received notices of the fact together with a request to renew their subscriptions at once. These notices were sent out early to forestall the competition of summer and vacation needs which sometimes cause renewals for THE MATHEMATICS TEACHER to be sidetracked for several months.

It is a great service to the business manager to know *now* what funds he has to work with and how many subscribers he can count on for the coming year. Much of the work of editing and producing a professional journal has to be arranged for months in advance of the actual issues. It is extremely difficult, if not impossible, to make these necessary arrangements unless the support of our members is assured. It is for that reason that members are asked to send in their renewals and $2.00 fees promptly.

Since there are no issues of THE MATHEMATICS TEACHER in June, July, August, and September, subscriptions which started with the October 1938 number close with the May 1939 number. Those who became members last fall, starting with the October number, are also urgently asked to make their renewals now for the coming year in the light of the considerations outlined above.

Some Factors Which Influence Success in College Algebra

By M. V. Marshall
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The Problem

In many colleges the course in freshman mathematics shows a higher mortality rate than any other course. Various reasons have prompted many investigators to study the reasons for this. Some have been interested in ascertaining if the study of mathematics required special abilities; some have been interested in predicting success or failure; some have sought more exact bases for student counselling; and some have been interested in obtaining more exact knowledge of the factors conditioning success or failure. The study here reported is more concerned with the last named purpose than it is with the others.

The Procedure

Measures of three different sorts were made on two classes ($N=44$) of freshmen in college algebra. Knowledge of high school algebra was measured by the Columbia Research Bureau Algebra Test (hereafter referred to as "Achievement"). A number of other studies have used average mark in high school mathematics, average mark in high school studies, and rank in senior class in high school as measures of students' preliminary knowledge. It was felt that, in view of the variations in the marking standards of different high schools, an objective test of students' algebraic ability would be a more reliable index of their preparatory background. It is obviously a measure of an ability more closely related to college algebra than the average high school mark or a student's rank. General academic intelligence was measured by the American Council Psychological Examination. This measure has been popular among investigators of this general problem. And, thirdly, specific mathematical intelligence was measured by the Iowa Algebra Aptitude Test (hereafter referred to as "Aptitude"). These three measures were obtained at the beginning of the course. A student's accomplishment in the subject was measured by the professor's term grade (hereafter referred to as "Grade"), a mark running upwards of 600 points (Range: $52-628$) which was obtained by totalling the student's grades in tests and examinations. It is probable that this mark was obtained in nearly as objective a manner and yields as reliable results as the other three tests. The professor of mathematics who cooperated in the study follows a system of marking that reduces personal judgment to a minimum. Questions are either right or wrong, but if partial credit is given a rule is formulated which is applied to all the papers. Subjectivity is eliminated as far as is humanly possible. From these four measures coefficients of correlation were computed.

Discussion of Results

Zero order correlations:

Grade with Aptitude: .557
Grade with Achievement: .788
Grade with Psychological Exam.: .492

The best single index of a student's grade is his results on a test of achievement in high school algebra. The correlation between the grade and the aptitude test is appreciably higher than that between the grade and the psychological examination but the difference is not striking.

Multiple correlations:

Grade with (Achievement and Psychological Examination): .808
Grade with (Achievement and Aptitude): .795
Grade with (Aptitude and Psychological Examination): .579
Grade with (Achievement and Aptitude and Psych. Exam.): .841

The three variables combined yield a

high correlation with the professor's grade. The two correlations in which the achievement in high school algebra was combined with a measure of general or specific aptitude, are practically identical and are both high. On the other hand, the correlation of the grade with the two variables measuring aptitude, the psychological examination and the algebra aptitude test, is very appreciably lower. This last point is in harmony with the tentative conclusion derived from the zero-order correlations, i.e., that achievement in high school algebra is a much more significant index of success in college algebra than a test of general or specific aptitude.

Partial Correlations:

Variables:
1. Professor's Grade
2. High School Algebra Achievement
3. Psychological Examination
4. Algebra Aptitude Test

$r_{13.2} = -.30$
$r_{14.2} = -.182$
$r_{34.2} = \ \ .134$

The two negative coefficients are of interest. With previous knowledge of algebra held constant the professor's grade and the psychological examination correlate −.30, and with previous knowledge of algebra held constant the professor's grade and the algebra aptitude test correlate −.182. This suggests unreliability of the professor's grade, but it has been shown above that this is likely to be very small, if present at all. Another, and a much more important, suggestion arising from the statistical facts is that, in an advanced course in algebra, a good previous knowledge of algebra is more important to success than either general academic intelligence or specific algebraic aptitude. This is not only highly significant because of its implications but is confirmed by the two previous conclusions regarding the importance of achievement as an index of success. The fact that the negative correlation in the case of the aptitude test is smaller than that in the case of the intelligence test is worth noting.

The relationship between the two tests of aptitude when algebraic knowledge as a factor is held constant is positive but its small size indicates that the two tests are not measuring the same thing. The correlation between them when the effect of the algebraic knowledge was not ruled out was $r_{34} = .673$, which, because it is not high, confirms the relative independence of the two variables, and, because it is as high as .673, suggests that the two tests of aptitude involve a common factor, possibly mathematical skill.

Achievement and Psych. Exam.: .77
Grade and Psych. Exam.: .492

Two other cases of zero-order coefficients will be considered. Both the achievement test and the professor's grade are measures of algebraic knowledge. The lower correlation of the professor's grade of achievement with the test of general intelligence may be due to unreliability of the grade. But a more likely cause is a difference in bases of marking. The Columbia Research Bureau Algebra Test consists of two parts: Part I dealing with the mechanics of algebra has a maximum score of 22, and part II dealing with problems has a maximum score of 43. The relative weight of computations to problem solving is about 1 to 2. In the the professor's mark a much greater weight is given to mathematical computational skill and relatively less weight to problem solving, which is more a matter of general intelligence. Consequently the grade given by the professor might be expected to have a lower correlation with the test of general intelligence than the algebra achievement test.

Grade with Aptitude: .557
Achievement with Aptitude: .793

The higher correlation of the aptitude test with achievement may be accounted for by the aptitude test being a test of achievement as well as of aptitude. As far as the writer has been able to ascertain the makers of the test have not shown that it is not a test of achievement. But the

disparity of the correlations given above can also be accounted for on the basis of the argument of the preceding paragraph, i.e., that the professor's grade is avowedly and consciously restricted almost wholly to measuring skill in algebraic computation and minimizes the weight given to exercises which make demands on general intelligence such as problem solving and similar *applications* of algebraic skill.

Conclusions

A few of the more important conclusions may be recapitulated in brief form:

1. In an advanced course in algebra a good knowledge of elementary algebra is probably a better basis for success than good general academic aptitude or specific mathematical aptitude.

2. The three variables, achievement in high school algebra, algebraic aptitude, and general intelligence, are here arranged in order of their correlation with achievement in college algebra.

3. Algebraic computation and problem solving are different abilities and should not be confused as measures of achievement in the subject.

4. A combined index of high school achievement, algebraic aptitude, and general intelligence yields a very high correlation with achievement in college algebra.

For Basic Drill in Arithmetic, What Norm or Average Is Satisfactory?

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A 75% grade is the most common passing mark; in some places it is as low as 65% or 70%; in others, as high as 80%, but 75% is decidedly the mode. Of the 206 schools in Odell's study, 3/4 of them have 75% as the passing mark.[1] With a 75% passing mark, since the range of scores will, no doubt, go up to near 100% for a few, there may be a class average of 85% or slightly more. What does an 85% class average mean in terms of mastery?

In a study[2] of the corrective load in the fundamentals of arithmetic in grades 4, 5, and 6, (1124 children), in a town near Boston, class averages in the four fundamental processes, as measured by very simple tests,[3] ranged from a low of 44.72 in short division in grade 5 in the October testing, to a high of 90.80 in subtraction and in short division in grade 6 in the March testing. Table I gives a summary of the averages for the four processes in the three grades in October, 1937, and again in March, 1938.

Many of the averages of Table I appear to be quite satisfactory, and they are, as far as averages go. In addition, five of the six averages for grades four, five, and six, are at 80 or above, or, in other words, are "high averages." There are other 90, and above, averages in subtraction and in short division respectively, in the March testing in the sixth grade.

However, these averages do not tell the entire story. What would we see if we were to spread the scores from 100 down? This spreading, or distributing, is done in Table II.

Table II shows for each of the three grades in the March testing, the distribution of the number of children receiving each score from 100 down to 0. The scores at the left, running from 100 down to 0 in steps of 4, are accounted for by the fact that in each of the tests used, there were 25 examples. To the right of the scores, under the different processes, is shown for each grade under process, the number of children who received the score shown at the left. It may be noted in passing that the slight discrepancy in totals for the different processes is due to the variation in attendance on the different days on which the tests were given.

At the bottom of Table II are shown the mean scores for the March testing, corresponding to similar means in Table I.

It is now possible to study Table II in order to see what spread of scores occurs for any particular mean. Let us examine those where the mean is 80 or above. The 84 mean for the fifth grade children in addition shows scores ranging down to 32. Yet anyone who notices the simplicity of this particular addition test[4] might reasonably expect that fifth grade children should make perfect scores. As a matter of fact, only 48 of the 379 children, or 13%, do make the perfect score.

In a similar way, the spread over any average may be noted. Sixth grade children in addition, with a mean of 90.04,

[1] Odell, C. W., "High School Marking Systems," *School Review*, XXXIII, May, 1925, pp. 346–354.

[2] Hanley, G. L., "Corrective Load in the Fundamentals of Arithmetic in Grades 4, 5, and 6," Master's Thesis, Boston University Graduate School, 1938.

[3] Wilson, G. M., *The Inventory and Diagnostic Tests in Arithmetic*, Spaulding-Moss, Boston, 1937.

[4] The Wilson Addition Process (AP) Test, Spaulding-Moss Company, Boston, 1937.

TABLE I

Brings Together the Mean Scores for Each Process for the Three Grades Tested, Showing a Comparison of the Means Found in the October and March Tests.

	Grade 4		Grade 5		Grade 6	
	Oct.	Mar.	Oct.	Mar.	Oct.	Mar.
Addition	65.36	80.00	80.88	84.00	87.96	90.04
Subtraction	45.84	74.00	75.12	87.20	84.28	90.80
Multiplication	—	—	57.60	75.20	81.65	80.40
Short Division	—	—	44.72	76.80	70.96	90.80
Long Division	—	—	—	—	61.76	79.80

have scores ranging down to 60 and only 53 of the 350 children, or 15%, have perfect scores.

The 87.00 mean for fifth grade children in subtraction shows scores running down to 4, which means that that particular child successfully solved only one of the 25 examples. 103 of these fifth grade children out of 404, or 25%, made the perfect score. This was in March, after considerable corrective work. Certainly no one would claim that the spread of scores such as here appears, is satisfactory for fifth grade children in the simple subtraction material of the Wilson Subtaction Process Test.

The reader may, in a similar way, figure the percentage of children in any of the grades of Table II, in any of the processes, who made a perfect score. Likewise, the spread of scores down to unacceptable depths may be noted.

TABLE II

Shows the Distribution of Scores from 100 Down to 0, for Each of the Grades 4, 5, and 6, for All of the Four Processes, for the March Tests.

	Addition			Subtraction			Multiplication		Short Division		Long Division
Grades	4	5	6	4	5	6	5	6	5	6	6
Scores											
100	21	48	53	50	103	104	34	23	91	96	63
96	23	64	68	40	79	84	35	30	56	68	57
92	38	44	78	32	61	53	45	45	42	59	58
88	35	34	60	20	37	44	43	49	31	33	26
84	46	44	36	25	23	20	25	44	27	23	22
80	31	38	23	15	22	10	31	31	12	22	20
76	34	33	12	12	16	14	40	33	12	13	11
72	18	25	7	11	11	10	24	32	19	4	9
68	16	10	4	12	14	3	24	16	12	6	11
64	13	15	5	11	10	3	24	10	11	5	7
60	9	5	4	11	4	2	14	12	8	6	17
56	10	6	—	11	5	3	11	—	6	11	10
52	5	5	—	12	3	2	11	6	3	4	5
48	2	1	—	3	1	1	2	2	11	4	7
44	6	2	—	6	5	4	4	—	4	4	5
40	3	4	—	6	1	4	6	5	8	3	4
36	—	—	—	3	2	—	6	4	3	2	2
32	—	1	—	8	2	—	4	1	6	—	10
28	—	—	—	4	—	—	1	—	3	2	2
24	—	—	—	5	2	—	4	—	4	—	3
20	—	—	—	5	2	—	4	—	4	—	1
16	—	—	—	4	1	—	3	1	3	—	2
12	—	—	—	9	—	1	1	—	2	1	1
8	—	—	—	3	1	—	1	—	2	1	—
4	—	—	—	1	1	—	—	—	1	—	—
0	—	—	—	1	—	—	—	—	1	—	4
Total	317	379	350	316	404	361	374	350	388	356	330
Mean	80.00	84.00	90.04	74.00	87.20	90.80	75.20	80.40	76.80	90.80	79.80

Since the averages (means) of Table II are reasonably high, as averages go, it is impossible to escape the thought that we may have been using averages to cover up the tragedy of errors in connection with the simplest and most fundamental of the work in arithmetic. Studies indicate that the four fundamental processes cover fully 90% of adult figuring.[5] Therefore, mastery of these processes is certainly the first task for the teacher of arithmetic in any grade above the third where they have not been previously mastered. (That is, addition and subtraction above grade 3, multiplication and short division above grade 4, and long division above grade 5.)[6]

At this point, reference to another study may be an advantage in helping to decide the meaning of an average and whether or not better scores than those indicated in this particular study are possible. A study of the corrective load in arithmetic in 15 towns and cities near Boston was undertaken and completed as a W.P.A. project in 1936. This study is briefly reported in the 1937 Yearbook of the American Educational Research Association.[7] The tables included in this report spread in a fashion similar to Table II shown above. For instance, one of the tables shows the distribution of scores on the Addition Process Test for grade six. This shows, for City A, a mean of 87.93, in addition. However, this mean appears below a distribution of scores ranging from 100 down to 40. Only 9% of the children fall at 100.

The story as just indicated for City A of the W.P.A. study referred to, is practically duplicated in 13 other cities near Boston. Scores in addition in grade 6, run down to 60, 48, 36, even to 0. The percentage of perfect scores does not range higher than 19%. In other words, 14 of the 15 cities in said W.P.A. study show for the sixth grades, distributions very similar to those shown in Table II above. The results in addition are very similar for grades 7 and 8.

However, there is one city in which sixth grade children have an average in the addition test, of 99.34. In this city, it will be interesting to note the complete spread of sixth grade scores in addition. 89% of the scores fall at 100, 7% at 96, 2% at 92, and 2% at 88. This city, in the seventh grade in addition, made a mean score of 99.58, 90% of the scores falling at 100, and 10% at 96.

The natural question, at this point, is What did the one city, City O, do to make scores decidedly better than those made by the other fourteen cities? There are reasons, and they're fairly simple, but it is not the purpose of this article to discuss these reasons.[8] The superior work in this city is referred to in order to point out the fact that approaches to the perfect scores in the simple tests of the fundamental processes are easily possible. The challenge for the perfect score has been explained more fully elsewhere.[9]

A few of the best of the 34 teachers in the study referred to on the first page of this article carry the suggestion of the possibility of superior results, when the distributtion of scores is critically examined.

One fifth grade teacher in this study showed in the March testing, 62% of her children at 100%, 35% at 96. The other scores spread down to 92, 88, and 84; at each of these points, 1% of the scores fell.

[5] Wilson, Guy M., "A Survey of the Social and Business Usage of Arithmetic," Teachers College Contribution to Education, #100, 1919. (Supported by later studies by Wise, Woody, Charters, Bobbitt, et. al.)

[6] Chapter on Arithmetic, Fourth Yearbook, Department of Superintendence National Education Association, 1926.

[7] Wilson, Guy M., "Corrective Load in the Fundamentals of Arithmetic in Grades 6, 7, and 8." 1937 Official Report of the American Educational Research Association.

[8] City O has followed the 100% Drill Plan in the fundamentals of arithmetic consistently for several years.

[9] Wilson, Guy M., "The Challenge of 100% Accuracy in the Fundamentals of Arithmetic," Educational Method, 15: 92–96, November, 1935. (Less fully reported in the 1935 Yearbook of the American Educational Research Association, pp. 70–74.)

This particular teacher appeared to grasp unusually well, the idea of the perfect scores, as well as the teaching and corrective procedures necessary to secure them.

This is hardly the place to note the reasons why teachers almost uniformly secure low scores in simple drill material. However, it is commonly agreed that some of the reasons are that drill starts too soon (Doubtless there should be no drill, as such, in grades one and two.); that the drill procedures are applied to many processes which are so little used as not to justify drill; that the teaching is not sufficiently well done (Many teachers do not know how many addition facts there are.); and that there is not the necessary review and follow up in the grades above the original teaching grade.

There are many considerations involved in a perfect score for all children at the appropriate grade levels which need not be taken up at this time. The chief purpose of this article has been to point out that averages (means), unless they are practically at the 100% point, can be quite deceptive. Teachers must learn to distribute their scores, to note what each individual has done and then to teach each child according to needs until perfect mastery has been obtained in the fundamentals of arithmetic. In time we shall learn at what grade levels drill in the fundamentals of arithmetic for perfect mastery, should be undertaken, and the teaching procedures for doing it efficiently and economically. Certainly no one will contend that the low mastery, as shown by the spread of scores in Table II, is satisfactory.

Any adequate criteria of drill should undoubtedly include the following: 1. It is not drill unless it is used very frequently in life, approaching daily usage. 2. It is not drill unless it must be used in identical form. 3. The drill load should be small enough to make success in mastery and retention an easy possibility. An impossible load means discouragement, confusion, and defeat. 4. The only acceptable score on simple useful drill material, is the perfect score.

Thus, whether we approach this question of satisfactory norms or averages from the standpoint of the subject, or the child, or a set of drill criteria, we arrive at the same conclusion. The subject and its functioning in life are best served by perfect mastery of the most needed processes. The child and his mental hygiene are best served, not by partial learning and confusions, but by perfect mastery and the confidence which accompanies it. A set of drill criteria urges the same, viz., perfect mastery of a sizable load of the most useful subject-matter.

Since, in any count, the processes of multiplication, addition, and subtraction, stand at the head of the list, these are the first processes for perfect and permanent mastery, followed at some distance, by fractions in halves, fourths, and thirds, and by long division. These basic tool processes are of concern not only in the teaching grade, but in any grade above the teaching grade until perfect mastery has been attained by *each pupil*. The teacher must cease to rely upon class norms or averages; she must learn to distribute her scores in order that she may see each individual. The only acceptable average in simple drill material is 100, or the average resulting from a perfect score by each child in the group. This is possible with good teaching.

National Council Members!

Don't forget the summer meeting of the National Council of Teachers of Mathematics to be held at San Francisco in connection with the N.E.A. See page 180 for the announcement of the meeting.

◆ THE ART OF TEACHING ◆

The Laws of Signs in Multiplication

By JAMES H. ZANT
Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma

IN A RECENT issue of THE MATHEMATICS TEACHER[1] a method of teaching the law of signs in multiplication was suggested and defended. The suggested procedure was essentially as follows: in the first type problem (3×5) the pupil was led to formulate the fundamental definition of multiplication by a positive whole number as a process of finding the sum of three equal groups; in like manner the second example (3×-5) is disposed of, that is, the pupil obtains the answer by adding three minus fives. In neither case, however, is an attempt made to get the pupil to formulate an explicit general definition of multiplication by a positive whole number.

The third case (-3×5), as we all know, is more difficult. Here Mr. Fulkerson introduces the commutative law for positive numbers and has the pupils make the tacit assumption that the law also holds for multiplication by a negative number. Thus $-3\times 5=5\times -3=-15$. "The rationalization of the process involved in the fourth example (-3×-5) is a *somewhat*[2] more difficult procedure" as Mr. Fulkerson says. The reader is referred to the original article; it defies my feeble powers of summarization.

Why not face the issue squarely and have the pupils make their definitions and assumptions explicitly? Multiplication by a positive whole number is already understood by ninth grade pupils. They are perfectly willing to define it as a short process of adding groups of the same size. This idea takes care of the first two types of problems (3×5) and (3×-5). It is probably worth while, however, to have the class make an explicit definition of multiplication by a positive whole number.

What is the meaning of multiplication by a negative whole number? This is a new idea to the ninth grade pupil and requires a fundamental definition. Any other approach is mere subterfuge and must always remain unconvincing to a clear-thinking pupil. Multiplication by a negative whole number may be defined straight from the shoulder and with some reasonableness as successive subtractions of the multiplicand. It seems that this is one of the first places where the mathematics teacher should seek to impress on the pupil the fundamental meaning of the science of mathematics. It consists of undefined terms, definitions, and postulates, with theorems developed from these. As the pupil proceeds with the study of the subject, it will be necessary for him to widen his horizon from time to time. He must make a new assumption or a new definition in order for him to be able to proceed into a wider field of mathematics and to apply his knowledge to useful areas. Probably it would be wise at this point to stress the fact that this particular definition of multiplication by a negative number is made so that the commutative law which applies to multiplication by positive numbers will also apply to multiplication by negative numbers. However, the pupils should get the idea that we make this kind of a definition merely because it is con-

[1] Elbert Fulkerson, "Teaching the Laws of Multiplication," THE MATHEMATICS TEACHER, XXXII: 27–29.
[2] Italics mine.

venient and not because it is a logical necessity.

The laws for the other two types of problems (-3×5) and (-3×-5) can easily be deduced from the foregoing definition. Minus three times five means that five is subtracted three times and we thus arrive at minus fifteen, using any of the numerous ideas of negative numbers. Minus three times minus five demands no elaborate rationalizing process. It means that minus five is subtracted three times and subtracting a negative number is equivalent to adding a positive number. The pupils arrive at the positive fifteen much more convinced of its reasonableness than by other more involved rationalization processes.

With the best method of developing such processes it may be doubted that the pupils obtain a full or adequate realization of their meanings on first acquaintance. An adequate understanding of an idea probably comes only after repeated views of the topic from various angles. Certainly a mathematician should not make the generalization that a class has a clear-cut notion of the complete process of multiplication with positive and negative numbers simply because the first pupil called on stated the law of signs for multiplication. It is easily possible that the pupil might have acquired the rule before the discussion, or the teacher might, as we are prone to do, have called on the brightest boy in the class or any one of a dozen other things might have been true. One of our aims in teaching algebra should be to present the material in such a way that the pupils will acquire the habit of thinking critically on any topic. This demands careful teaching but it also demands a logical, straight from the shoulder presentation by the teacher.

Summer Meeting of the National Council!

The National Council of Teachers of Mathematics is having its annual summer meeting jointly with the N.E.A. in San Francisco on July 3, 4, and 5. Stimulating and authoritative speakers and discussion leaders are being provided. Among them are such well-known personalities as E. R. Hedrick, Paul Hanna, the Rev. W. G. Gianera, H. M. Bacon, Earl Murray, Miss Emma Hesse, and Mrs. Alta Harris.

There will be a general session on Monday afternoon at which Dr. Hanna and Dr. Hedrick will speak. On Tuesday afternoon there will be a session on arithmetic and one on senior high school mathematics. On Wednesday afternoon will be a session on junior high school mathematics and one on junior college and teacher training problems. On Wednesday noon, there will be a discussion luncheon at International House on the University of California Campus for which reservations should be made with Miss Edith Mossman, 2229 Derby Street, Berkeley, California as soon as possible. Price 88 cents, tax included.

Dr. H. B. Bacon, Stanford University, is in charge of local arrangements. Miss Emma Hesse, University High School, Oakland, California, is in charge of publicity and will be able to provide details of information. The Palace Hotel will be headquarters. See the May issue of The Mathematics Teacher for the final program.

◆ EDITORIAL ◆

WHAT ARE WE GOING TO DO ABOUT IT?

In an article called "The Defeat of the Schools" in *The Atlantic* for March 1939, Professor James L. Mursell of Teachers College, Columbia University, says*

Most people—certainly most laymen—would be apt to think that the chief business of the schools is to give pupils at least a modest working knowledge of the subjects of the curriculum. Not a few students of education, it is true, consider that this is a misconception, and that the true purpose of schools is to bring about an adjustment to social demands for which the various subjects are at best only means. Nobody, however, who surveys the conventional working apparatus of courses of study, textbooks, recitations, examinations, and marks can have much doubt that in practice the schools are making the mastery of the curriculum an end in itself. Whether in theory they ought to or not, this in fact is what they are manifestly trying to do.

But they are not succeeding. About that there is no room for theorizing. Nor is their failure sporadic, and confined to a few places where management is unusually bad. It occurs almost universally, in the cities and in the country, in large institutions and in small. The schools go through an elaborate, costly, and exasperating set of motions called teaching natural science, foreign languages, English, and so forth. Yet what is contained in textbooks and syllabi obstinately refuses to transfer itself to the minds of the patrons and stay there. In the grand struggle to get subject matter off the page and into the head, the schools are suffering a spectacular and most disconcerting defeat.

This, I am aware, is a formidable statement. But it is made on formidable evidence. The recent investigation carried on by the Carnegie Foundation in the colleges of Pennsylvania, issued under the title *The Student and His Knowledge*, which has caused considerable furore over the ignorance of college seniors, is only a drop in the bucket. For proof of the defeat of the schools does not depend on one investigation alone, or on a dozen, or a score, or a hundred. We have here nothing less than the consistent testimony of thirty years of enormously varied research in education.

Since about 1910 many thousands of investigations have appeared, dealing with almost every conceivable aspect of school work. Comparatively few of them explicitly and directly consider the efficiency of the schools in terms of how much subject matter they manage to induce the pupils to learn. Yet a great many, without particularly setting out to do so, throw a most startling light upon the results of education. Taken in the mass, they add up to a consistent, coherent, and extremely impressive body of testimony, pointing towards one conclusion: whatever the goods which our schools are delivering, they are not what one might expect to find in packages labeled science, history, foreign languages, English, and so forth.

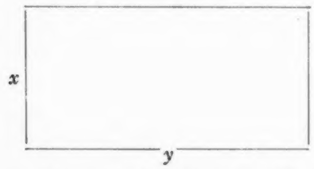
Suppose we call the roll of the principal subjects, asking in each case what a person ought to learn if he is to have anything worth calling an elementary working mastery of it, and also to what extent this is really being acquired.

Let us start with mathematics. Here we have the most precise and powerful instrument for abstract thinking and analysis devised by man. This, indeed, is the very essence of the subject. Mathematics is a technique of thinking, and if you have not learned to think in its special language you just have not learned mathematics at all. Such thinking may not be very intricate or advanced; but, all the way from simple arithmetic to differential equations and beyond, it is the same kind of process. That process is quite different from doing sums, because when one is set a sum one is told either directly or by the most obvious sort of hint whether to add, subtract, multiply, divide, or what not, and all one has to do is to follow certain rules and remember certain tables, which clearly is routine rather than thinking. Essentially the job of every mathematician all the way from a first grade child to an Einstein, is to take hold of situations and disentangle them by the techniques of the science. A beginner may not be able to deal with very complicated situations, or to carry the disentangling very far. But he must go some appreciable distance along this road—he must be able to decide what treatment the given situation requires, whether addition or multiplication or something else—before he can claim anything that can possibly be called mastery of mathematics.

Now we know beyond peradventure that a very large percentage of children studying mathematics in school go hardly a step in this direction. For instance, the educational authorities in Iowa have carried through an ambitious project known as the "Every-Pupil Testing Program." Part of that program was an examination for mastery of the material taught in ninth-grade algebra. In constructing this examination it was clear to the experienced workers that they must use only very simple items, or almost every pupil would be defeated. (An ominous consideration at the start. Inexperienced test-makers tend strongly to expect far too much.) Sixty-two such items were assembled, samples being the following: "Write a formula for the perimeter of the rectangle.

* Quoted by permission of *The Atlantic*.

Write a formula for the area of the rectanguler.



A dealer sold a suit of clothes for $42, making a profit of 20 per cent on the cost; how many dollars did he make?" Simple enough, one would say; yet genuine problems, because they require not the mere recalling of knowledge, but its application to the analysis of situations. Yet it appeared that half of the large number of pupils taking the test, all of whom had taken algebra, could do no more than ten of the items. Other results showed that actual working knowledge of the ostensible content of the algebra course was being acquired in only the most meagre degree.

Again, another investigation was carried on in which a test in elementary mathematics was given to a large number of children. When they were simply told to perform certain operations, such as factorization, reduction of fractions, the "solution" of easy equations, and the like, 1000 out of 1200 succeeded. But when problems were set up, which not only were simple, *but involved no operations other than those which these children had shown they could perform*, only about 300 succeeded. What shall it profit anyone to be able to do a certain kind of sum on instruction but unable to use his skill in a problematic situation? Can we say that such a person has any genuine mastery of mathematics?

Very similar results are reported for geometry. Pupils may be able to repeat the proofs of theorems, or to recite definitions and postulates; but this is no evidence that they can deal with "originals"—that is, with problems for the attack upon which theorems and definitions are only tools. Once more we ask in what sense a person has "mastered" a set of tools which he cannot use. If you feel inclined to say that one cannot learn a proof without thought and understanding, consider this: it has been shown again and again that even very small accidental changes in the proof of a theorem will destroy the ability of many pupils to repeat it. These children know how to prove that the square on the hypotenuse of the right-angled triangle *ABC* is equal to the sum of the squares on the other two sides. Good! Now call the triangle *PQR*. A surprising percentage of your group will be rendered helpless. Have they really learned and understood the proof at all?

Moreover, geometry does not stay learned. College freshmen, all of whom had studied the subject within the past two years, were given a test designed for high-school students: 86 per cent of these freshmen were below the high-school norm; 50 per cent did worse than 75 per cent of the high-school students; and 26 per cent did worse than nine-tenths of the high-school students. Also, they were conspicuously weak in applying geometric reasoning to problematic situations—that is, in the *use* of the geometric equipment they had labored to acquire. A subject which has been grasped and understood does not fade from the mind so easily and fast.

Other investigations have turned the spotlight on the intimate processes of pupils who are tackling the job of dealing with simple mathematical problems. Put pupils up against a situation requiring mathematical thinking, and what happens? Do they carry through intelligent methods of attack, rational in themselves even when slips in computation result in wrong answers? Do they experiment and analyze and try out alternative procedures in some sort of planned sequence? The author of one important study finds that few of them seem to reason at all, and that reflective thought is not evoked. "Instead, many of them appear to perform almost random calculations upon the numbers given. Where they do solve a problem correctly the response seems to be determined largely by habit."

Still another body of disconcerting evidence is at hand. One would naturally suppose that the longer a person studied a subject, the more mastery of it he would gain. Yet, when tests of mathematical competence which do not simply duplicate the situations and routines of textbook and courses are applied, it has often been found that there is little of such gain with time. Apparently pupils are going through textbooks and courses but not effectively gaining in mathematical insight and mastery. Everything comes together in an inescapable conclusion. Mathematics is being energetically taught, but it is not being learned to a degree that would, judged by commonsense standards, seem worth while.

In natural science the situation is much the same. Teachers of physics in twenty-eight schools began to have misgivings about the efficiency of their work and the amount of physics actually penetrating the minds of their pupils. Obviously a person might be able to recite fairly well on assignments prepared *ad hoc* the night before, or do fairly well on examinations closely related to the course, and still have no independent mastery of the subject as such. So the teachers constructed a test which, on the basis of their pooled judgment, represented what one ought to know when he had covered the two topics of mechanics and heat. The results were illuminating, not to say startling. The majority of pupils fell far below expectation. Such questions were asked as: "A stone is dropped and takes two seconds to hit the ground; from what height is it dropped? How much work is done to raise two pounds one yard?" These particular problems had never been set, but everyone was supposed to have the knowledge and technique needed to solve them. Inability to answer could only mean lack of mastery of the subject as contemplated in the courses; yet this appeared very generally. Also, it was found that student's performance on the independent test had very little relationship to his rating in the course in physics. It was one thing to give correct answers

under the special promptings of assignment, textbook, and recitation, but quite another to show a mastery of physics as such.

A few years ago a comprehensive test to reveal competence in chemistry, including in its scope, mechanical operations, the use of formulae and equations, and general information, was given to a group of students in high school, and to college students who had completed freshman chemistry. Let us begin with the showing of the college students. Remember that all of them had finished freshman chemistry. Some had taken the subject in high school, and others had not. But on the test there was no significant difference in the performance of these two groups. Apparently the time spent studying the subject in high school as a preliminary to taking it in college was an almost total loss. University teachers might easily think this a severe reflection on high-school chemistry. So indeed it is. But they should not boast too soon. For those who had taken chemistry in high school and had not yet gone further showed up just about as well on the test as those who had taken the subject both in high school and as freshmen in college. Two years of the subject did not give much better results than one, whether that one year was spent in high school or in college.

Moreover, with both physics and chemistry it has been shown that failure is most conspicuous in the learning of those techniques and principles of analysis which are the chief values of these subjects, and on which their practical application principally depends. A smattering of unrelated facts, rather than an insight into integrated, functioning laws, is what is acquired. And even such facts are insecurely learned.

Quite recently investigators have been raising a new question, more fundamental than any I have discussed so far. Does the study of science, including mathematics, teach scientific thinking? Just conceivably this might happen even without the student's gaining any very extensive or exact information, or any very firm grasp upon techniques and methods of analysis. Of course it is much harder to tell whether a person is able to think scientifically than whether he knows certain facts or is able to use certain techniques, so results here are apt to be less reliable than elsewhere. Still, the attempt has been made rather frequently, with great care and seriousness, and the answer to the question is: No! There is practically no evidence that science, as taught in school, makes one more careful about hypotheses, more willing to suspend judgment, more open-minded towards alternative views, more able to distinguish truth from hokum. It does not, so far as we can tell, successfully promote these or any other typical virtues of the scientific mind.

He then discusses the case for the so-called language arts—foreign languages, English, and the social studies, and history—with as pessimistic an outlook as he gives for mathematics and science. Moreover, he finds that even among college seniors the range of knowledge for many of them is no wider than sophomores and some of them are no better informed than high school seniors. Professor Mursell then turns to the question of *transfer of training*. He says:

The other body of experimental results, however, is quite a different affair. These are the investigations dealing with what is known as *transfer of training*. Probably all of us have at some time been told that a subject which seems useless and futile for its own sake is still worth studying because it helps with some other subject. Latin is said to help with other foreign languages, and to improve one's English. Grammar is thought to enable one to write and read better. Elementary mathematics is supposed to be an aid to physics. Geometry trains one in reasoning. Such claims are quite familiar. What one learns in one subject is said to *transfer* to others, and so to help in mastering them.

The hypothesis of transfer has been one of the chief topics of educational and psychological research. Something like one hundred and seventy published investigations have been devoted to it. Their general and accepted testimony is that very limited transfer takes place. Students who have taken Latin show up no better in other languages than those who have not; nor do they read better, write better, spell better, or use larger vocabularies in their native language. It has been shown again and again that work in English grammar has almost no value as an aid to the accurate and competent use of the English language. Persons who have spent a great deal of time studying grammar do no better either in composition or in reading than others who have had little or no grammatical training. Teachers of physics frequently complain that when pupils enter their courses they seem to forget all the mathematics they ever knew, and research backs up such grumblings. As to the argument that geometry teaches people to reason better about things in general, it is preposterous, and advanced without a shred of evidence.

The cumulative results of this long list of investigations are authoritative and accepted. They are summarized in almost every elementary textbook on educational psychology, and are one of the commonplaces at teachers' gatherings. Practically every professionally trained teacher in America has heard the news that transfer of training does not take place, though without a very critical or adequate insight into the evidence.

When one studies a subject, any benefits there may be accrue within the subject itself and not somewhere else. Such is the ascertained fact. (I state it broadly, and without various, not unimportant, qualifications.) But the proper inference from it, the moral of the tale, is hardly ever pointed out. *Transfer of training ought to*

take place. Its failure to do so is a reproach to teaching. When pupils cannot use their mathematics in a physics course, something must be wrong. Perhaps they never really learned their mathematics in the mathematics course! Latin, with its close affiliation with other languages, ought to help with French, Spanish, and German, not to mention English. When this fails to happen, it is a reflection upon the teaching of Latin. Again, English style is, in a sense, applied grammar. So when we prove that learning grammar does not improve English expression—and this has been proved to the hilt—the inference must be that grammar was pretty badly learned.

Lack of transfer is not a law of nature or a fiat of the Almighty. It is an indictment of teaching. Learn Latin as it should be learned, and it will help you with other languages. Acquire a real grasp of mathematical thinking, and it will not fail you when you tackle physics. The investigations on the transfer of training simply go to swell the great volume of evidence that the schools, in their attempt to generate adequate and worth-while masteries in the subject-matter fields, are meeting with defeat.

Finally, in trying to decide what material should be taught in the schools, he says "We must make a selection." He then continues,

How do we make it? We make it on the basis of nothing more intelligent or reassuring than long tradition. The material taught in the school goes far back through the years, some of it to a remote antiquity. Of course, like every other tradition, it has altered somewhat with the passage of time, but slowly and superficially. It has a sort of independent life of its own, which is highly resistant to change. Textbooks and syllabi tend to be based on previous textbooks and syllabi. Teachers tend to purvey what they themselves have learned. The pattern of the basic curriculum is extraordinarily rigid, and has never been critically reconstructed from the ground up, except in a few favored institutions. In the main the schools continue to teach it for no better reason than that it has always been taught.

Now how much confidence can we have that a body of content so selected is really worth learning? Surely very little! Can we say it is a balanced and representative sampling of the best and finest that the human spirit has achieved and is achieving? By no means. In the standard curriculum there is some fine gold, but also an unconscionable quantity of dross. If we wanted to give our convenient inquiring friend, the Man from Mars, an idea of the best in human culture, we would hardly hand him a set of school texts and syllabi. Why should we do it with Johnnie and Susie? Or can we say that what we offer is vitally related to the interests, concerns, and needs of young Americans? Again the answer is: No! Nothing of the sort has been considered in making the selection. Indeed, it is notorious that this material has been assembled in advance, without any reference to the kind of people who are supposed to learn it.

But this is a fatal weakness. If we ourselves can have no great confidence in the schools, how can we expect Johnnie and Susie to believe in it? And if they have no authentic sense of the importance and value of the things they learn, they cannot—literally they *cannot*—learn them well. For the human mind is not naturally docile. It is capable of amazing feats of resistance and rejection, beneath a tame and dutiful exterior. It assimilates into its life and makes its own only those things which, for some genuine reason, seem to matter. Everything else stays on the surface and soon evaporates. This is no recondite scientific discovery, but simple common sense. Everybody knows it from his own experience, though not everybody drags it into consciousness as a guiding principle. Watch Johnnie and Susie at work on some hobby. Then contrast them as they work at school tasks. The difference? Obvious!

Here, I insist, is where our trouble starts. We set up a body of material which, in the nature of the case, must be mastered not because of its intrinsic and manifest appeal, but under some kind of duress. Learning under no urge except external duress, however, is contrary to all natural tendency. Resistances are set up which frustrate the process, no matter how "good" or docile the learner seems. These are the forces which defeat the schools.

Is there a way out? Of course there is. The first necessity is to abandon the *idée fixe* of a standard body of content which everyone must learn. People young and old learn what matters to them, what seems of genuine moment to them. Whatever fails to come with the authentic impact of reality and need is automatically and fatally rejected. In a very genuine sense each one of us makes his own curriculum; for the only curriculum that matters is the one a person carries in his head, rather than the one in the textbooks and syllabi.

Hence the great necessity is for far more flexibility in our whole treatment of children in the schools, and above all for flexibility in what we ask them to master. Many reformist and experimental schemes have this as their controlling principle, and they succeed in so far as they put it into effect. Teachers should be free to bring to their pupils those portions and aspects of subject matter which are of immediate and living concern. They should not be doomed to keeping a rigid lock step, or to covering a predetermined area.

This does not in the least mean that we shall stop teaching mathematics, natural science, literature, and the like, and substitute current events, wood carving, and cookery. It only means that not everybody will learn the same mathematics, natural science, and literature, and that people will not always learn them in the same internal order and sequence. We seem to have heard that one man's meat is another

man's poison. Why not apply this hackneyed wisdom here? Any person's cultural meat—the culture which he is able to assimilate, and which nourishes him—depends upon his present life interests, his status, his needs, his concerns. And our schools must be so organized that it will become possible to choose for a given individual or a given group at a given time those elements of culture which will indeed provide nourishment.

Will this prevent pupils from mastering the "logic" of mathematics, or natural science, or social science? They are not mastering it now! Strange to say, they are not mastering it precisely because we present it to them as a logic. Mathematics, for instance, is a technique or tool of thought. That is its essence, its logic. But we do not learn to use this tool by first studying its inner structure and organization up to a certain point, and then applying it. We learn to use it by actually using it, in no matter how haphazard and fumbling a fashion, upon problems which we really want to solve. So with all the other disciplines. Let us handle our subject matter as something which is alive, and its logic will take care of itself.

To organize the schools in terms of flexibility rather than rigidity is no small or easy task. It calls for much revision of conventional procedures and instrumentalities. Yet it can be done, and in fact the work is going on apace. For the benefit of those conservatives who may think that the way to get children to learn more in a school is a return to the good old days of high pressure and rigid requirements, I project one more nugget of ascertained fact. It has been shown that the experimental schools actually produce better subject-matter learning than the conventional schools, with pupils of equal ability. Carry such tendencies further, and we have good reason to expect still more satisfactory results. What we contemplate is, to be sure, a breach with some of our most adamantine traditions and customs. But we have before us the problem of an intelligent rather than a stupid approach to the task which is of such supreme importance in a democratic society—the task of bringing to the individual his birthright of culture. It is a task in which we cannot afford to, and need not, accept defeat.

Who will say that, on the whole, these charges by Professor Mursell are not true? The problem for mathematics teachers is to decide the best course of action to take in view of the facts. It is to be hoped that THE MATHEMATICS TEACHER may be the medium of getting some of the more modern organizations of material and better methods of presenting this material to the pupils before the teachers of mathematics in this country.

The article referred to above does not give detailed advice as to what should be done about the situation in which we find ourselves and it tells us little if any that we do not already know. The situation is like that which is reported to have stimulated Mark Twain to remark "Everybody complains about the weather, but nobody does anything about it." It is high time that something be done to improve instruction in the great fields of knowledge of the secondary schools.

W. D. R.

How Old Is Ann?

I WAS very much interested in the problem, "How Old is Ann?" (See page 125 of THE MATHEMATICS TEACHER for March 1938) and solved it in one unknown, I enjoyed seeing the solution in five unknowns and thought some one might enjoy mine.

After one realizes that if x is Ann's earliest age mentioned then $2x$ will express the difference in their ages, it is easy to get the following relationships from the problem by working back on it.

Here is the problem and solution: The combined ages of Mary and Ann are 44 years. Mary is twice as old $2(9x/2-2x)$ as Ann was $(9x/2-2x)$ when Mary was half as old as Ann will be $(9x/2)$ when Ann is three times as old as Mary was $(9x)$ when Mary was three times as old as Ann. $(3x)$. How old is Ann?

From this one gets the equation:

$$2\left(\frac{9x}{2}-2x\right)+2\left(\frac{9x}{2}-2x\right)-2x=44$$

(I did not reduce these fractions and quantities for reasons of clarity.) The solution gives $x=5\frac{1}{2}$ and from that one readily finds Mary to be $27\frac{1}{2}$ and Ann $16\frac{1}{2}$.

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IN OTHER PERIODICALS

By Nathan Lazar
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1. Austin, Charles M., "Motivating Mathematics." *School Science and Mathematics*, 39: 134–147, February, 1939.

The following are some of the topics treated: (1) the purpose of plane geometry, (2) the mastery of subject matter, (3) geometric facts—sum of the angles of a triangle and other polygons, ratio and proportion, Pythagorean Theorem, rigidity of a triangle, area formulas, similarity, measuring inaccessible distances, principles of construction, navigation, designs and decoration, architecture, nature, (4) precise and accurate use of words, (5) nature of proof, (6) indirect proof, (7) discovering new truths, (8) symmetry of form.

There are also sections on "Algebra," "The Number System," and "Mathematics Interprets Our Economic and Scientific Environment for the Individual." An interesting feature of the last topic is a list of 108 varied questions the answers to which are mathematical.

2. Courant, Richard, "Mathematical Education in Germany before 1933." *American Mathematical Monthly*, 45: 601–607, November, 1938.

After a brief sketch of mathematical education in Germany during the 19th century, the author gives a detailed and very interesting account of the conditions prevailing there in this century prior to 1933.

The following are some spectacular but isolated excerpts: "The requirements of the state board examinations for high school teachers were excessively high; they called for full-fledged scholars and probably could never be enforced even approximately. . . . A considerable number of the famous German mathematicians of the nineteenth century started their careers as more or less successful teachers in high schools. . . . At the University of Berlin in some years more than 600 students registered for a single course in number theory, and about as many in calculus and analytic geometry. At Göttingen courses on the theory of function of a complex variable had an attendance of up to 330 students. In highly specialized and advanced courses, 50–100 students could often be found, many of whom were also preparing for a teaching job. . . . There were no official examinations and no marks through the years between high school and the state board examination (except examinations for tuition scholarships). It happened that candidates, after five years of attending advanced classes and having had little contact with instructors and fellow students, came up for examination, and to the dismay of both sides it turns out that they knew literally nothing about mathematics and had stupidly wasted their years at the university. The absolute freedom for the student to select his courses and to conduct his studies to his own taste was sometimes a handicap for the majority, although of the highest benefit to the top group. . . . The state board examinations required written theses, and very often these theses originated in advanced courses or seminars and were the nuclei of doctoral theses. Subject and standards of these state board theses varied greatly. . . . Not infrequently these theses contained some modest original contribution, and in most cases they at least proved that the student had studied some field with real understanding. Sometimes, it is true, results were very poor and unsatisfactory."

3. Craig, H. V., "Invariants and Elementary Mathematics." *National Mathematics Magazine*, 13: 176–182, January, 1939.

The purpose of this paper is to call attention to the fact that some of the extremely simple notions of invariant theory may be employed to advantage in undergraduate teaching. It is also pointed out that one of the objectionable features of elementary mathematics, as presented in the traditional manner, is the practice of demonstrating general theorems by means of figures and ignoring the question of the generality of the figures. The writer believes "that invariants and vector analysis should play conspicuous rôles in geometry and trigonometry. In algebra, determinants should be developed through the e—systems of tensor analysis. This would not only provide an excellent introduction to the technique of tensor analysis but would provide the background for the definition of the cross product in terms of the three e—symbols."

4. Hedrick, E. R., "The Function Concept in Elementary Teaching and in Advanced Mathematics." *The American Mathematical Monthly*, 45: 448–455, Aug.–Sept., 1938.

This article is the first to appear in a new department entitled "Mathematical Education."

The author points out the persistence of the function idea through every stage of mathematical development, and touches upon the advanced mathematical studies which center around the function concept.

The important rôle that the function concept occupies is traced through the entire range of mathematics—from arithmetic, through algebra, geometry, trigonometry, college mathematics, the calculus, functions of a real variable, and functions of a complex variable.

The article concludes with the remark that "this is the one theme which tends to unify all of mathematics and to permit its integration with life and with science. Would that all teachers of mathematics, all students of mathematics knew this truth! Would that all professed leaders of education were aware of it."

5. Hope-Jones, W., "Simplicity and Truthfulness in Arithmetic." *The Mathematical Gazette*, 23: 7–25, February, 1939.

The presidential address to the (British) Mathematical Association, January, 1939.

It would be futile to give anything but a list of the topics discussed: checkability, cancelling, the improper fractions superstition, the unitary method, pi, the extra-figure superstition, what makes an approximation "correct"?, remainders, percentage, young methods, addition and subtraction, and the use of tables.

6. Logsdon, Mayme I., "Geometries." *The American Mathematical Monthly*, 45: 573–583, November, 1938.

A very illuminating exposition of a difficult, though familiar, topic.

7. Mutch, Heber R., "The Teaching of Elementary Products and Factors." *School Science and Mathematics*, 39: 147–148, February, 1939.

Products of binomials by binomials are usually divided into four or five types. The author points out the pedogogic defects of such classification and proposes another one based on two types: type I includes all binomial products whose first or second terms are equal, including sign; type II includes those in which neither the first nor the second terms are equal. The advantages of this procedure are pointed out.

8. Price, G. B., "A Program for the Association." *The American Mathematical Monthly*, 45: 531–536, October, 1938.

A program is proposed for the Mathematical Association of America based on the following two aims:

I. To foster adequate, high quality instruction in high schools, and in colleges and universities.

II. To assimilate the results of mathematical research and to interpret mathematics to scientists and the public.

To accomplish Aim I, a program is suggested consisting of two parts: (a) conferences and symposia at the national meetings to achieve that aim; (b) a department in the *Monthly* to treat these matters. To accomplish Aim II, it is suggested that a series of lectures be instituted to be called *Herbert Ellsworth Slaught Lectures*. The scope of these lectures is carefully pointed out.

A bibliography of twenty-one items is included.

9. Richeson, A. W., "Unpublished Mathematical Manuscripts in American Libraries." *National Mathematics Magazine*, 13: 183–188, January, 1939.

The author requested about 250 of the largest college, university, private, state, and municipal libraries throughout the country to give him a list with as much information as possible, of their mathematical manuscripts which were written prior to 1800. These lists were checked to determine whether or not they had been published at some time. The printed list of unpublished manuscripts contains a full description of each manuscript—authors, title, size and format, date of composition, and also the name of the library where it is available.

10. Smith, David Eugene, "Possible Boundaries in the Early History of Mathematics." *American Mathematical Monthly*, 45: 511–515, October, 1938.

The author describes the boundaries of a decade ago of our knowledge of the early history of mathematics, and then the way in which they have been extended. He anticipates a further extension in the future.

"We may therefore hope that some of the younger scholars of our generation will follow their predecessors and continue the study of the clay tablets which have revealed so much ancient history in the field of mathematics. This is the most fruitful field at the present moment. There is next the study of papyri. In spite of the fact that these sources are not as permanent as the clay tablets they are usable as long as they last and may be valuable. The wall paintings and reliefs have proved of great value, especially when including numerals and their applications. There is also the possibility of the traditions of the Orient in the manipulation of numbers which may reveal something worth considering in number theory."

NEWS NOTES

Report of the Work of the Affiliated Organizations of the National Council of Teachers of Mathematics, February, 1939

A. Activities.

1. On October 5, letters were sent to all state representatives explaining to them the purposes and obligations of affiliated clubs and urging them to secure affiliations within their own state. An application blank was enclosed.
2. On October 20, lists of obligations and advantages of affiliation were sent to the presidents of the affiliated clubs.
3. On January 10, letters were sent to all club presidents describing the Cleveland meeting and making suggestions in reference to plans for delegates. An information sheet was included. On this, the clubs were requested to report the names of the present officers, the name of the delegate to the Cleveland meeting, detailed report of programs and suggestions for helping the clubs.
4. About two weeks previous to the Cleveland meeting, postal cards were sent as reminders to those who had not returned the information blanks.
5. About the first of February, the booklets prepared by the secretary *Twenty Questions Asked and Answered Concerning the National Council of Teachers of Mathematics* were sent to the club presidents.
6. Miscellaneous correspondence.
7. Four clubs voted to affiliate while Miss Lane was attending a meeting of the club as guest speaker. Other clubs were encouraged to affiliate.

B. Number of Clubs in Each State.

1—Alabama	2—Missouri
1—Arkansas	2—Nebraska
2—California	*1—New Mexico
2—Colorado	8—New York
1—Georgia	*1—North Carolina
3—Illinois	1—North Dakota
1—Indiana	3—Ohio
4—Iowa	3—Oklahoma
2—Kansas	1—Pennsylvania
*2—Kentucky	1—South Carolina
1—Maryland	2—Tennessee
1—Michigan	1—Texas
2—Minnesota	1—West Virginia
1—Louisiana-Mississippi	1—Washington, D.C.
1—Washington-Idaho	1—Wisconsin

* Each of these states has a club which has applied for affiliation but it has not been granted. We are attempting to get the difficulty corrected and the records completed.

The four clubs which have not completed their affiliation are: Mathematics Section, State Teachers Association of North Carolina; Mathematics Section, New Mexico Education Association; Mathematics Department of the Northern Kentucky Education Association.

Number of clubs now affiliated—51.

Number of new clubs which have affiliated during 1938–1939—13.

Number of clubs which have applied for affiliations not yet granted—3.

C. The Clubs Which Have Affiliated during 1938–1939.

Arkansas—Mathematics Section, Arkansas Education Association;

Indiana—The Mathematics Section of the Indiana State Teachers Association;

Iowa—Mathematics Section of the Northeast District of the Iowa State Teachers Association, Mathematics Section of the Southeast District of the Iowa State Teachers Association, Mathematics Section of the South Central District of the Iowa State Teachers Association;

Missouri—Mathematics Section of the Northeast Missouri District Teachers Association;

New York—Nassau County, Mathematics Teachers Association, Suffolk County, Mathematics Teachers Association;

North Dakota—Mathematics Section of the North Dakota Education Association;

Ohio—Mathematics Club of Greater Cincinnati;

Texas—Mathematics Section of the Texas State Teachers Association;

Wisconsin—Mathematics Section of Southwestern Wisconsin;

Washington, D. C.—Benjamin Banneker Mathematics Club.

D. Suggestions for Increased Help to the Clubs.

1. The suggestion to the clubs that they elect a secretary for a period of several years with whom the correspondence could be carried on by the national officer. In most cases, it is now necessary to contact new officers each year through the previous officers.

2. That we make an attempt this year to help clubs find speakers. The request for suggestions of ways we could be more helpful brought back the almost unanimous appeal for knowledge of speakers who were planning to be somewhere near the local club and who could be secured to address the club.

   The following plan is suggested to be attempted for next year:

   a. Blanks may be sent to the National Council officers and others upon which to give the place and date of speaking as soon as arrangements are being made.
   b. The news of future speaking dates might be printed in THE MATHEMATICS TEACHER along with the name of the speaker and the location of the club.
3. Club officers may be asked to report on the information blank the topics and speakers which have been exceedingly helpful and inspiring.
4. A campaign may be carefully planned for selling the report of the Commission on the Place of Mathematics in Secondary Education through the clubs.

E. News Items from Clubs.

The Wichita (Kansas) Mathematical Association has a program for this school year which includes six meetings. On March 23, Dr. Congdon will come from Nebraska University to address the club.

The Illinois Association of the National Council of Mathematics Teachers had as the theme of their conference: "Instructional Procedures for the Slow and Bright Pupil." Seven Illinois teachers gave ten minute talks on various aspects of this theme at one session. At another session, Professor Raleigh Schorling of the University of Michigan talked on "Techniques of Teaching Mathematics to the Slow-Learning Pupil."

Many of the clubs report that "The Preliminary Report of the Commission on the Place of Mathematics in Secondary Education" has been reported on and discussed either by the entire group or in smaller groups.

The Mathematics Section of the Eastern Division of the Colorado Education Association has some good ideas to pass on to others. In the spring they met jointly with the Rocky Mountain Section of the Mathematical Association of America. The teachers were asked to bring exhibits of work done in various mathematics classes. Dr. Kempner of the University of Colorado, who was at the time also the president of the Mathematical Association of America, addressed the group on "Trends in Mathematics and Mathematics Teaching." They had two other meetings in the fall. They also have an executive council of sixteen members, consisting of the three officers, three from the colleges of Colorado, seven from the senior high schools, and three from the junior high schools, which meets four times a year to discuss matters tending to foster interest in mathematics teaching of both local and national importance. They also publish a bulletin which is sent to all members, to all high school principals and superintendents in the Eastern Division, and to all instructors of mathematics in the colleges of the state. To finance this are yearly dues of one dollar per member. When the treasury can stand it, they pay part of the expenses of a delegate to the convention of the National Council.

President H. C. Christofferson addressed the Mathematics Section of the Middle Tennessee Teachers Association on "Geometry as a Way of Thinking."

Section 19 (Mathematics) of the New York Society for the Experimental Study of Education was addressed by Professor W. D. Reeve on "The Mathematical Education of Present Day Germany." He showed motion pictures of his recent trip there. Professor W. L. Schaaf spoke on "The History of Mathematics as an Integral Part of the Curriculum."

The Cleveland Mathematics Club has a printed program for the school year with four meetings scheduled in addition to entertaining the National Council in February.

The Range Mathematics Club of Minnesota plans to have a meeting in one of the underground mines near Eveleth in April. In November, Mr. Fred H. Weck, meteorologist, U. S. Department of Agriculture from Duluth, spoke and illustrated his talk with movies and slides. In February, they had sound film pictures as the major feature of the program. These included "The Play of Imagination in Geometry."

The Minneapolis Mathematics Club had some of the administrators of their city schools on their programs and also W. E. Peik, Dean of the College of Education of the University of Minnesota. At one meeting they had a panel discussion on "The Present Curriculum in Use" with six speakers on special aspects of this theme.

The Detroit Mathematics Club has a printed directory of the club for 1938–1939 and includes the program for five meetings. Speakers from Michigan and Martha Hildebrandt from Maywood, Illinois, are addressing these meetings this year.

Maurice L. Hartung of the University of Chicago will address the Kansas Association of Teachers of Mathematics on April first. This

association publishes a bulletin of news to mathematics teachers four times a year.

The Mathematics Section of the Iowa State Teachers Association has a curriculum committee of twenty members now functioning for the third year. It reports annually to the group and last year published a bulletin which was sent to all mathematics teachers in the state. Martha Hildebrandt helped to make the annual meeting a success. An exhibit, brought by teachers of the state, was very popular.

The Mathematics Section of the Southern Division of the Colorado Education Association sponsors an annual luncheon meeting at the time of the state meeting. This is a worthwhile project for clubs because it helps teachers to become better acquainted.

Miss Elizabeth Dice, representative in Texas, is to be congratulated for securing the affiliation of the Texas State Association of Mathematics Teachers. Other state representatives deserve credit also for affiliations of clubs within their states. Still other representatives report that clubs will soon vote for affiliation.

RUTH LANE
Second Vice-President of the National Council of Teachers of Mathematics.

Dr. Ferdinand von Lindemann, German mathematician and scientist, died in Munich on March 7 at the age of eighty-six.

Dr. von Lindemann was the first to prove the mathematical impossibility of squaring the circle, and for this achievement received the prize of the Prussian Academy of Sciences. He published many books and papers dealing with mathematical problems and contributed frequently to German, French, and English periodicals. He collaborated with H. Poincare in two volumes on science and mathematics. At the age of seventy-six he conducted research into ancient Egyptian weights and measures.

From 1893 until his retirement as Professor Emeritus, Dr. von Lindemann was privy counselor and a full professor at the University of Munich. He was rector there in 1904–05, and at the University of Koenigsberg in 1892–93, where he was a full professor from 1879. Previously he had been an assistant professor at the University of Freiburg, and had taught mathematics at the Universities of Erlangen and Wuerzburg.

Dr. von Lindemann studied at the Universities of Goettingen, Erlangen, Munich, London and Paris. He was a Doctor of Philosophy, and received honorary degrees from several German universities.

The Men's Mathematics Club of Chicago and the Metropolitan Area met March 17 at Central Y.M.C.A. The program centered around the topic "How Evanston Is Attempting to Meet the Problem of Individual Differences." Members of the Evanston group discussed the following topics: "Outline of the Ten-Year Effort," "Classification of Freshmen," "The General Mathematics Course of Two Years," "My Experience with Two Sections of General Mathematics," "Basic Mathematics," "Freshman Algebra," "3-Algebra," "3-Algebra-S," "Selection of 3-Algebra-S Students," "Trigonometry," and "Solid Geometry."

The Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics met jointly on March 3 and 4, at Louisiana State University, University, Louisiana. The program was as follows:

Joint preliminary meeting, 2:00 P.M., March 3. Address of welcome by Dean Fred C. Frey, Louisiana State University.

Louisiana-Mississippi Section of the Mathematical Association of America, 2:30 P.M., March 3, J. F. Thomson, Tulane University, Chairman; W. V. Parker, Louisiana State University, Secretary. The following papers were given: "Squares Escribed and Inscribed to a Triangle," B. E. Mitchell, Millsaps College; "A Proof of the Addition Theorems of Trigonometry," W. L. Duren, Tulane University; "The Summation of Finite Series," Albert Farnell, Louisiana State University; "The Least Super Sphere of a Set," H. T. Fledderman, Loyola University; "Cremona Involutions Determined by a Pencil of Surfaces," F. C. Gentry, Louisiana Polytechnic Institute; "Equations with Coefficients in a Modular Field," Charles Hopkins, Tulane University; "Tchebycheff Approximation for Decreasing Functions," C. D. Smith, Mississippi State College; "A Problem in Minimum Values," F. A. Rickey, Louisiana State University; "Equations of Heat in the Fins of Air-cooled Engines," W. B. Brown, Mississippi Woman's College.

Joint banquet, Highland Hall, 7:00 P.M., March 3, J. F. Thomson, Toastmaster. Welcome by Dean C. A. Ives, Louisiana State University. Address, "Observations on the Study and the Teaching of Mathematics," E. T. Browne, University of North Carolina.

Meeting of Louisiana-Mississippi Branch of National Council of Teachers of Mathematics, 8:00 A.M., March 4. Herbert C. Ervin, Long Beach, Miss., Chairman. The following papers were given: "The Distance of the Stars," D. V. Guthrie, Louisiana State University; "Mathematics Clubs in High School," Mrs. H. L.

Garrett, Istrouma High School; "Visual Aids in Teaching of Geometry," Miss Jessie May Hoag, Jennings High School; "Sidelights in the Teaching of Mathematics," R. L. O'Quinn, Louisiana State University; "The New Organization of Freshman Mathematics Classes at Louisiana State University," J. P. Cole, Louisiana State University; "An Analysis of the Freshman Placement Test at Louisiana State University," H. T. Karnes, Louisiana State University; "The Selective Process in the High School Mathematics Program," A. C. Maddox, Louisiana State Normal College.

Meeting of the Louisiana-Mississippi Section of the Mathematical Association of America, 10:00 A.M., March 4. The following addresses were given: "A Formula in Installment Buying," P. K. Smith, Louisiana Polytechnic Institute: "Some Unit and Zero Identities," V. B. Temple, Louisiana College; "The Cevian Tetrahedron and Some of Its Remarkable Points," M. C. Wicht, Louisiana State University; "Quasi k-commutative Matrices," E. T. Browne, University of North Carolina.

A general discussion, business meeting, and election of officers closed the session.

W. H. BRADFORD
Secretary

The Oklahoma Council of Mathematics Teachers met in Tulsa, February 10, at 2 P.M. with James H. Matthews, Oklahoma City, as Chairman and Kathleen Bagley, Tulsa, as Secretary.

New officers elected were: President, Eunice Lewis, Sepula; Vice-President, Leslie Dwight, Shawnee; Secretary, Kathleen Bagley, Tulsa.

After the business session, members of the Council enjoyed a violin solo by Miss Sarah Burkhart of Tulsa. This was followed by a most helpful talk on "Remedial Instruction at the Secondary Level" by Professor Raleigh Schorling, of the School of Education, University of Michigan.

Professor Schorling spoke before the Oklahoma Section of the Mathematical Association of America the same morning on "The Place of Mathematics in General Education."

CARRIE MAE LITTLE
